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# CONSIDERATIONS IN ROMANIA FOR ESTIMATING POTENTIAL GDP

**Anca ANDREI, Ramona-Mihaela PAUN**

Academy of Economic Studies Bucharest, Romania
rmpaun26@yahoo.com

**Abstract.** *In this paper we present the methodology used for estimating NAIRU using filters and we also consider the posibility of a convex Phillips curve that captures an asymmetric relationship between inflation and unemployment and discuss some problems of estimates. Asymetry in Phillips curve (which is represented by the convexity of the curve) means that high unemployment has relatively limited effects in pulling inflation down, whereas low unemployment can be much more effective.*

***Keywords:*** *output gap, Kalman filter, NAIRU, asymmetric Phillips curve*

**Clasificare JEL**: *JEL*: C02, C51, C24, E24, E20

**Clasificare REL**: 8E

## 1. Introduction

There is no consensus among economists about the precise sharp of the Phillips curve as studies on US data lead to different results:

- convex sharp: Turner (1995); Clark et al. (1996); Debelle and Laxton, (1997);
- concave sharp: Eisner (1997) and Stiglitz (1997);
- linear: Gordon (1997).

The difference of opinion on the empirical evidence may imply high costs regarding economical results of political decisions.

The meaning of asymmetry is that the response of unemployment to output growth is different when the economy is expanding from that when the economy is contracting. The conventional specification, which encompasses symmetry, would let us believe that expansions and contractions in output have the same absolute effect on unemployment. This focus came at a time when economists started to be interested in asymmetry in business cycles in general, an idea that can be traced back to Keynes (1936), who suggested that downturns may be sharper than upturns.

Asymmetry in Okun's law is related to asymmetry in the Phillips curve, because the latter is a combination of Okun's relationship and the aggregate supply curve. While asymmetry in Okun's law means that downturns in the economy are more rapid and sustained in driving unemployment up than recoveries in bringing it down, asymmetry in the Phillips curve (which is represented by the convexity of the curve) means that high unemployment has relatively limited effect in pulling inflation down, whereas low unemployment can be much more effective. In other words, the Phillips curve is asymmetric if unemployment below NAIRU tends to result in increasing and eventually explosive inflation, whereas excess unemployment will have a diminishing effect, tailing away into insignificance.

## 2 Mehodology for NAIRU estimation using filters

*HP filter for NAIRU*

The statistical approach can be unifactorial or multifactorial. For a unifactorial approach we can use Hodrick-Prescott (HP) filter. In order to use this method to determine the unemployment deviation we first have to determine the long term trend component of unemployment $U_t^*$ by minimising the problem defined in ecuation (5):

$$\underset{U_t^*}{\text{Min}} \left\{ \sum_{t=1}^{T} (U_t - U_t^*)^2 + \lambda \sum_{t=2}^{T-1} (\Delta U_{t+1}^* - \Delta U_t^*)^2 \right\}$$

The adjustment of trend component depends on the selection of parameter $\lambda$ .

In general, $U_t^*$ determined by this approach is known as an estimate of NAIRU because it does not explicitly incorporate information on the structural variables that determine the natural rate of unemployment.

Empirical analysis revealed that the NAIRU obtained with HP filter (based only on current unemployment) is significantly correlated with changes in inflation, suggesting that even this simple approach leads to significant results.

A possible disadvantage of the HP approach is that estimates the deviation of unemployment, without appeailing to information about inflation. Multifactorial statistical approach such as Kalman filter has the advantage that estimates NAIRU (or potential output) as variable in time, in close correlation with the Phillips curve. The reason for using multifactorial techniques is that it employs more information in determining the NAIRU and potential output.

*Kalman filter for determining the NAIRU*

Below we present the method for determining the Kalman filter NAIRU:

$$\Delta\pi_t = \alpha(L)\Delta\pi_{t-1} - \theta(U_t - U_t^*) - \gamma(L)\Delta U_t + \xi_t \qquad \xi_t \sim N(0, H)$$

$$U_t^* = U_{t-1}^* + \eta_t \qquad \eta_t \sim N(0, Q) \text{ and } \operatorname{cov}(\xi_t, \eta_t) = 0$$

where: $\Delta$ is the first difference operator, $\pi_t$ is the annual inflation rate, $U_t$ is the unemployment rate, $U_t^*$ is NAIRU, $\eta_t$ and $\xi_t$ are the error termns.

Equation (2) is the generalised Phillips curve. Expectations are implicitely assumed in inflation dymanics. The second equation (3) specifies NAIRU, $U_t^*$, as a random walk process and the unknown terms are $\theta$ si $U^*$.

From equation (2) conclude that inflation increases when unemployment falls below NAIRU, influenced by additional effects from the unemployment rate changes due to past changes of inflation.

There are two sources of change in inflation in this model. The first source can be an inflationary shock due to exogenous events. The second can be the change of NAIRU itself. Kalman filter extracts the noise signal (the observed changes in inflation and unemployment) providing an estimate of the NAIRU.

Since the NAIRU is determined by structural factors that evolve gradually over time, some restrictions on the variation of the error terms (Q and H) are required, which are considered in the context of Kalman filter method. For simplicity, usually in practice, H is normalized to unity, leaving Q which is restricted (Gordon 1997).

*Univariate Kalman Filter*

The output equation is:

$$y_t = za_t + e_t$$

$$e_t \sim N(0, H)$$

where $y_t$ is the observable output variable, $a_t$ is the unobservable state variable.

$$a_t = Ta_{t-1} + u_t$$

$$u_t \sim N(0, Q)$$

The matrix H and Q are matrix of variations for the observable and unobservable variables.

The error terms $e_t$ and $u_t$ sare assumed to be serially independent.

The model for estimating the output deviation is:

- actual output identity:

$$y_t = y_t^* + gap_t \qquad (4)$$

- potential output equation:

$$y_t^* = y_{t-1}^* + \eta_t \qquad (5)$$

Output gap equation:

$$gap_t = \phi_1 gap_{t-1} + \varepsilon_t \qquad (6)$$

$y_t$ is the log of real GDP, seasonally adjusted, $y_t^*$ is the output potential, $gap_t$ is the output gap, $\eta_t$ and $\varepsilon_t$ represents shocks that are assumed to be independent and identically distributed (iid) with average zero and constant variation.

Equation (4) is an identity that shows the actual output as the sum of potential output and output gap. Equation (5) represents the potential output as a random walk process. Equation (6) defines the output gap as a AR(1) process.

We consider the dynamic system:

$$x_t = Za_t + e_t$$

$$a_t = Ta_{t-1} + u_t$$

in which we assume the observable output vector $x_t$ observabil and the unobservable state vector $a_t$ . We can rewrite equations (4) and (6) as:

$$[y_t] = [1 \quad 1]\begin{bmatrix} y_t^* \\ gap_t \end{bmatrix}$$

$$\begin{bmatrix} y_t^* \\ gap_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \phi \end{bmatrix}\begin{bmatrix} y_{t-1}^* \\ gap_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix}$$

where:

$$x_t = [y_t],\ Z = [1 \quad 1],\ a_t = \begin{bmatrix} y_t^* \\ gap_t \end{bmatrix},\ T = \begin{bmatrix} 1 & 0 \\ 0 & \phi \end{bmatrix} \text{ and } u_t = \begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix}.$$

We can estimate the system using Kalman filter with the maximum likelihood method.

## 3. Estimating output gap using production function method

Production function method is a standard multivariate method used for estimating potential output as a function of total factor productivity, capital and labor, all employed at their potential level.

Unlike HP filter, the production function method has the main advantage of providing usefull information regarding input contribution to potential output but the estimates depend on the techniques employed for input smoothening and requires longer time series.

For estimating potential output and output gap we consider the following Cobb-Douglas production function with constant returns to scale:

$$Y_t = A_t (K_t)^{\alpha} (L_t)^{1-\alpha} \tag{3}$$

where $Y_t$ represents real output, $A_t = e^{\delta + \eta t + \varepsilon_{yt}} = A_0 e^{\eta t + \varepsilon_{yt}}$ is total factor productivity (TFP), $K_t$ represents capital stock, $L_t$ is labor force, $\alpha$ and (1-$\alpha$) represents capital and labor contributions to output[1].

Liniarizing (3) yields:

$$\ln Y_t = \ln A_t + \alpha \ln K_t + (1-\alpha) L_t \tag{4}$$

For a given $\alpha$, the log value of total factor productivity $(tfp_t = \ln A_t)$ is derived from:

$$tfp_t = y_t - [\alpha k_t + (1-\alpha) l_t] = a + \beta t + \varepsilon_{y_t} \tag{5}$$

in which small letters denote log values for Y, K, L and $a = \ln A_0$.

The production function for potential output is:

$$Y_t^{pot} = A_t^{pot} \left(K_t^{pot}\right)^{\alpha} \left(L_t^{pot}\right)^{1-\alpha} \tag{6}$$

where $A_t^{pot} = e^{\gamma + \theta t} = e^{\gamma} e^{\theta t} = A e^{\theta t}$ represents the HP filtered total factor productivity and $K_t^{pot} = K_t c_t^{NAICU}$ is the potential capital stock corresponding to the capacity utilization rate that does not accelerate inflation (NAICU- Non Accelerating Inflation Capacity Utilization Rate) that is derived by HP filtering capital stock.

For potential labor we employ the equation that was proposed by Giorno etal (1995):

$$L_t^{pot} = L_t^S \left(1 - u_t^{NAWRU}\right) \tag{7}$$

where $L_t^S$ represents civil active population at time t filtered with HP filter and $u_t^{NAWRU}$ is the unemployment NAWRU rate (Non Accelerating Wage Inflation Rate of Unemployment) that is also HP filtered. Therefore, $L_t^{pot}$ corresponds to the number of people that could be employed if the unemployment rate would equal its natural rate given by NAWRU.

Considering the above mentioned notations, potential output can be written as:

$$Y_t^{pot} = A e^{\theta t} \left(K_t^{NAICU}\right)^{\alpha} \left(L_t^S \left(1 - u_t^{NAWRU}\right)\right)^{1-\alpha} \tag{8}$$

and the output gap is defined as the difference between real output and its potential divided by potential output:

$$output \quad gap\_FP = \frac{Y_t - Y_t^{pot}}{Y_t^{pot}} * 100 \tag{9}$$

The output gap can take positive values (when real output>potential output) and in this case the aggregate demand growth exceedes the aggregate suply growth. This could lead to inflation, thus we call it inflationary gap. If output gap values are negative, then we have a recessionary gap that could lead to deflation.

For estimating potential GDP and output gap we used STATA. Total factor productivity was calculated based on the value 0.65 for labor contribution to output acoording to the estimations provided by Dobrescu (2006: pp. 71).

The production function is:

$$Y_t = 1.97 e^{0.02t} (K_t)^{0.35} (L_t)^{0.65} \tag{13}$$

and potential output is calculated with:

$$Y_t^{pot} = 1.97 e^{0.02t} \left(K_t^{NAICU}\right)^{0.35} \left(L_t^S \left(1 - u_t^{NAWRU}\right)\right)^{0.65} \tag{14}$$

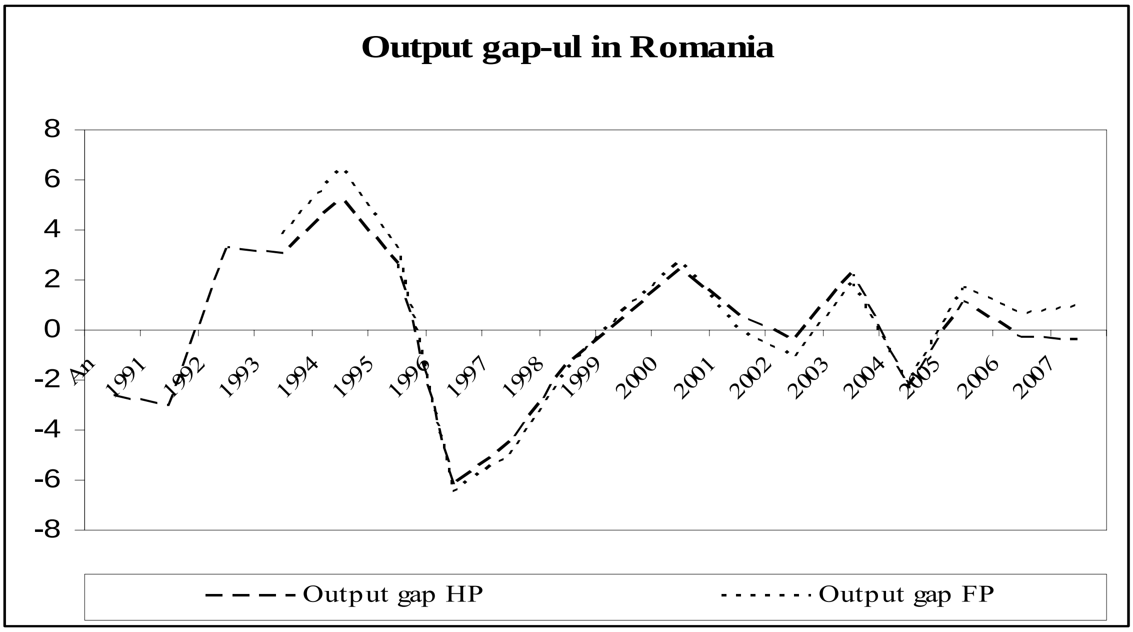


Figure. 1. Output gap with HP filter and production function method

In this paper we were interested in estimating potential output and output gap in Romania between 1991-2008 using an univariate method- HP filter and a multivariate method- the production function method. The production function is Cob-Douglas type and potential labor is determined based on NAWRU that is obtained with the method suggested by Elmeskov (1993).

Both methods provide similar results indicating variations within the range of -0.36% and 2% for potential output between 1994-2000 but beginning 2001 it will start to increase towards 5%-6.7%, between 2004-2008.

As for the evolution of output gap, we notice the existence of stronger gaps for the first years (expansionary between 1993-1996 and recessionis between 1997-1999) that are decreasing starting 1999 varying between -2.3% and +2.7% in 1999-2008.

## 3. The asymmetry of Phillips curves

The convex shape Phillips curve in terms of unemployment means that as the unemployment falls below its sustainable level, the upward pressure of inflation rises increasingly, on the margin. In this case, it is proven that the stabilization policies matters.

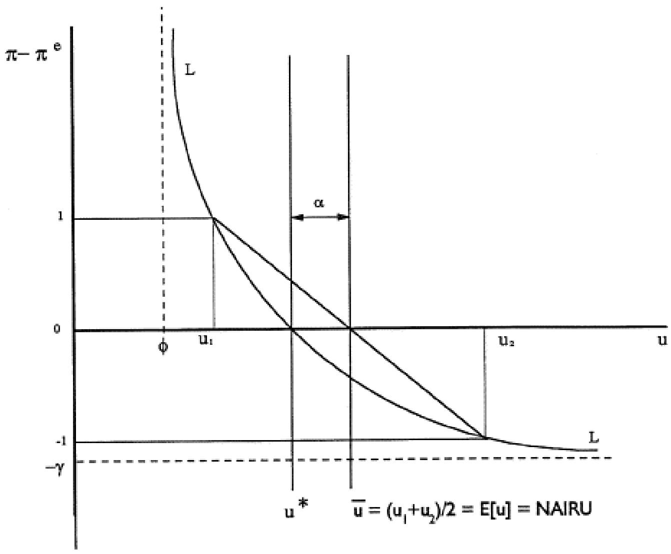

Figure 2: The implications of convexity on Phillips curve

In figure 2, the vertical axis shows the inflation, the horizontal axis shows the role of unemployment, *u*. Convexity means that the cyclical trade-off between inflation and unemployment worse on the margin as the latter is pushed below the point $u^*$ where $u^*$ is the deterministic NAIRU or DNAIRU.

DNAIRU is the level of *u* at which there is no systematic pressure for inflation to rise or fall, relative to expectations, in absence of shocks (hence deterministic).

$\bar{u}$ is average level of *u*, called "*the natural rate*", it is consistent with expectation equilibrium, usually named NAIRU, or the stochastic equilibrium rate. NAIRU is the level of *u* where there will be no acceleration (or deceleration) of inflation in the stochastic setting.

$$\bar{u};\ E[u] = \frac{1}{2}(u_1 + u_2) \quad (7)$$

α is the difference between the NAIRU and the DNAIRU. The size of *α* depends on the degree of convexity and the dispersion of *u*. In the convex case, the stabilisation policy has effects because could reduce the variability of *u* and lower its mean value.

The convex Phillips curve proposed by <u>*Debelle and Laxton (1997)*</u>, *Laxton ed al (2000) is:*

$$\pi_t = \lambda \bar{\pi}_t^e + (1-\lambda)\pi_{t-1} + \gamma(u_t^* - u_t)/(u_t - \phi_t) + \varepsilon_t^\pi \quad (8)$$

where:
$u^*_t$ is the DNAIRU;

$$\bar{\pi}_t^e = \left(\sum_{i=0}^{12} \pi_{t-i}^e\right)/12 \quad (9)$$

$\pi^e_j$ is the one-year-ahead expectation of inflation, held at *j*.

Parameter $\phi$ defines a lower bound on *u*, reflecting short-run constraints on how far rising aggregate demand can lower unemployment before capacity constraints become absolutely binding and inflationary pressure becomes unbounded. One can allow $\phi$ being time-varying:

$$\phi_t = MAX(0, \bar{u}_t - 4) \quad (10)$$

where $\bar{u}_t$ is a measure of trend unemployment..

$\phi_t$ is constrained to be zero when the trend unemployment rate is at or below 4%.

Using data from different articles: Cornelia Scutaru, Cristian Stanica ( 2004); Ciprian Turtureanu (2007); Elisabeta Jaba et al (2008) and from [www.insse.ro](http://www.insse.ro) , as well, we obtained the following (<u>*Debelle and Laxton (1997)*</u>, *Laxton ed al (2000)* estimated function:
Estimated convex Phillips curve for Romania

$$\pi_t = 0{,}305679\bar{\pi}_t^e - 0{,}203130\pi_{t-1} - 79{,}7840(u_t^* - u_t)/(u_t - \phi_t) + \varepsilon_t \quad (11)$$

Hyeon-seung Huh (2002) considers a standard Phillips curve augmented with an LSTAR (Logistic Smooth Transition Autoregression) component:

$$\Delta\pi_t = \sum_{i=1}^{p}\theta_i\Delta\pi_{t-i} + \sum_{i=1}^{q}\eta_i\Delta u_{t-j} + \left[\sum_{i=1}^{p}\theta^*_i\Delta\pi_{t-i} + \sum_{i=1}^{q}\eta^*_i\Delta u_{t-j}\right]F(z_{t-k}) + \varepsilon_t \quad (12)$$

where:

$\Delta$ is the first order difference operator, $\pi_t = 1/4\ln(CPI_t / CPI_{t-1})$ is a quarterly inflation at an annual rate; $u_t$ is the unemployment rate, and $\varepsilon_t$ is a disturbance term.

The logistic function is assumed to have the following form:

$$F(z_{t-k}) = (1 + \exp(-\lambda(z_{t-k} - c)/\sigma_z))^{-1} \quad (13)$$

where $F(z_{t-k})$ lies in the range between 0 and 1. The variable $z_{t-k}$ is switching indicator that represents the state of the economy, and the parameter ***c*** represents the threshold around which the dynamics of the model change. The parameter $\lambda$ is the smoothness parameter measuring how rapidly the transition between the regimes is processed. The parameter $\sigma_t$ is the standard deviation of switching variable $z_{t-k}$.

For estimation we used the function:

$$\Delta\pi_t = \theta_1\Delta\pi_{t-1} + \theta_2\Delta\pi_{t-2} + \eta_1\Delta u_{t-1} + \mu_2\Delta u_{t-2} + (\alpha_1\Delta\pi_{t-1} + \alpha_2\pi_{t-2})(1 + \exp(-\lambda\bar{u}_t / 2{,}305)))^{-1} + \varepsilon_t \quad (14)$$

where $\bar{u}_t$ is NAIRU trend, $\sigma_u = 2{,}305$ is standard deviation of $\bar{u}_t$, $\pi_t$ and $u_t$ having the signification above.

The estimated Phillips curve in that case is:

$$\Delta\pi_t = 0{,}329409\,\Delta\pi_{t-1} - 0.465129\,\Delta\pi_{t-2} + (0{,}036110\,\Delta\pi_{t-1} - 0{,}121287\,\pi_{t-2})(1 + \exp(-0.932652\,\bar{u}_t / 2{,}305)))^{-1} + \varepsilon_t \quad (15)$$

## Conclusions:

The first objective of this paper was to produce an estimate for potential output and output gap in Romania between 1991-2008 using a multivariate method- the production function method. The production function is Cob-Douglas type and potential labor is determined based on NAWRU that is obtained with the method suggested by Elmeskov (1993).

For the evolution of output gap, we notice the existence of stronger gaps for the first years (expansionary between 1993-1996 and recessionis between 1997-1999) that are decreasing starting 1999 varying between -2.3% and +2.7% in 1999-2008.

The second objective was to estimate a Phillips curve for Romania. Since the linear function did not produce a good estimate for Phillips curve, we used an assymetric analitical form of Phillips curve. We preffered the assymetric form because it yields better estimations and there were theoretical and empirical evidence that suggests nonlinearities in the Phillips curve also apply for Romanian data. We intend to use the estimated assymetric function for determining the optimal monetary policy rule.

[1] Assuming that the price of capital reflects its marginal productivity and wages reflect labor marginal productivity.

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