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## ABOUT SINGLE-PHASE VOLTAGE RECTIFIERS OPERATION

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**Abstract:** Some issues regarding the analysis of single-phase voltage rectifier operation will be presented in this paper. The switch is an uncontrolled-type, a diode in this case, which is considered as an ideal one. The expressions of the parameters characterizing the rectifier operation are computed in the paper. Their variations depending on load are also discussed.

**Keywords:** rectifier, diode, single-phase

### 1. INTRODUCTION

The schematic of an angle-phase voltage rectifier is presented in figure 1.

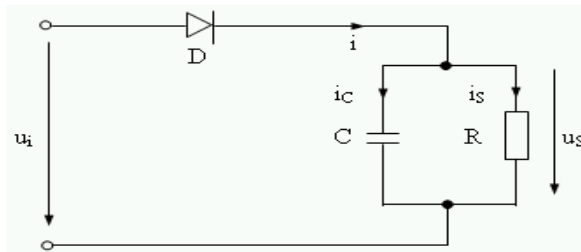


Fig.1. Single-phase voltage rectifier with diode

The steady-state signals expressions are the following [1]:

The diode is "on":  $\omega t \in (\alpha_p, \beta)$

$$\begin{cases} i_S = \frac{U_M}{R} \sin \omega t = I_M \sin \omega t \\ i_C = \omega C U_M \cos \omega t \\ i = U_M \sqrt{\frac{1}{R^2} + (\omega C)^2} \sin(\omega t + \xi) = \\ = I_m \sin(\omega t + \xi) \end{cases} \quad (1)$$

The diode is "off":  $\omega t \in (\beta, 2\pi + \alpha_p)$

$$\begin{cases} i_S = -i_C = I_{S\beta} e^{-(\omega t - \beta) \text{ctg} \xi} \\ u_S = R i_S = U_M \sin \xi e^{-(\omega t - \beta) \text{ctg} \xi} \end{cases} \quad (2)$$

where:

$$\begin{cases} \beta = \pi - \xi \\ I_{S\beta} = \frac{U_M}{R} \sin \beta = I_M \sin \xi \\ \sin \alpha_p = \sin \xi e^{-(\pi + \alpha_p + \xi) \text{ctg} \xi} \end{cases} \quad (3)$$

The following notations were used:

$$\begin{aligned} \text{tg} \xi &= \omega RC \\ Z &= \frac{R}{\sqrt{1 + (\omega RC)^2}} \\ I_M &= \frac{U_M}{R} \\ I_m &= \frac{U_M}{Z} = \frac{I_M}{\cos \xi} \end{aligned} \quad (4)$$

## 2. CIRCUIT ANALISIS

### 2.1 The Voltages and Currents Average Values

According to [2], the expressions of the average values of the circuit signals are the following:

$(U_m)_{\alpha_p}$  - average value of the output voltage:

$$(U_m)_{\alpha_p} = \frac{U_M}{2\pi} \left[ \cos \alpha_p + \frac{1}{\cos \xi} * \left[ 1 - \sin^2 \xi e^{-(\alpha_p + \pi + \xi) \text{ctg} \xi} \right] \right] \quad (5)$$

$(U_{\text{def}})_{\alpha_p}$  - RMS value of the output voltage:

$$(U_{\text{def}})_{\alpha_p} = \frac{U_M}{\sqrt{2\pi}} * \sqrt{\pi - \xi - \alpha_p + \sin(\alpha_p + \xi) \cos(\alpha_p - \xi) + A} \quad (6)$$

where

$$A = \frac{\sin^3 \xi}{\cos \xi} \left[ 1 - e^{-2(\alpha_p + \pi + \xi) \text{ctg} \xi} \right]$$

$(I)_{\alpha_p}$  - average value of the current flowing through the diode:

$$(I)_{\alpha_p} = \frac{1}{2\pi} \frac{U_M}{R} [\cos(\alpha_p + \xi) + 1] \quad (7)$$

$(I_S)_{\alpha_p}$  - average value of the load (R) current:

$$(I_S)_{\alpha_p} = \frac{I_M}{2\pi} \left[ \cos \alpha_p + \frac{1}{\cos \xi} * \left[ 1 - \sin^2 \xi e^{-(\alpha_p + \pi + \xi) \text{ctg} \xi} \right] \right] \quad (8)$$

Limit cases:

For pure resistive load,  $\omega RC = 0, \xi = 0$  it results:

$$\alpha_p = 0^0, \beta = 90^0 \quad (9)$$

$$\begin{aligned} (U_m)_{\alpha_p} &= \frac{U_M}{\pi} \\ (U_{\text{def}})_{\alpha_p} &= \frac{U_M}{2} \\ (I)_{\alpha_p} &= \frac{1}{\pi} \frac{U_M}{R} = \frac{I_M}{\pi} \\ (I_S)_{\alpha_p} &= \frac{I_M}{\pi} \end{aligned} \quad (10)$$

For pure capacitive load,  $\omega RC = \infty, \xi = \frac{\pi}{2}$

it results:

$$\alpha_p \approx 90^0, \beta \approx 180^0 \quad (11)$$

$$\begin{aligned} (U_m)_{\alpha_p} &= U_M \\ (U_{\text{def}})_{\alpha_p} &= U_M \\ (I)_{\alpha_p} &= 0 \\ (I_S)_{\alpha_p} &= 0 \end{aligned} \quad (12)$$

### 2.2 Rectifier Parameters

According to [2], the following parameters are defined:

The periodicity factor:

$$k_m = \frac{(U_m)_{\alpha_p}}{U_M} = \frac{1}{2\pi} \left[ \cos \alpha_p + \frac{1}{\cos \xi} * \left[ 1 - \sin^2 \xi e^{-(\alpha_p + \pi + \xi) \text{ctg} \xi} \right] \right] \quad (13)$$

The  $k_e$  factor:

$$k_e = \frac{(U_{\text{def}})_{\alpha_p}}{U_M} = \frac{1}{\sqrt{2\pi}} * \sqrt{\pi - \xi - \alpha_p + \sin(\alpha_p + \xi) \cos(\alpha_p - \xi) + A} \quad (14)$$

unde

$$A = \frac{\sin^3 \xi}{\cos \xi} \left[ 1 - e^{-2(\alpha_p + \pi + \xi) \text{ctg} \xi} \right]$$

The shape factor:

$$k_f = \frac{k_e}{k_m} \quad (15)$$

The wave factor:

$$k_u = \sqrt{k_f^2 - 1} \quad (16)$$

The graphic representations of the four parameters variation as functions of load



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$\xi = \text{arctg}(\omega RC)$  are shown in the figures below.

The graphing was performed by using Mathcad.

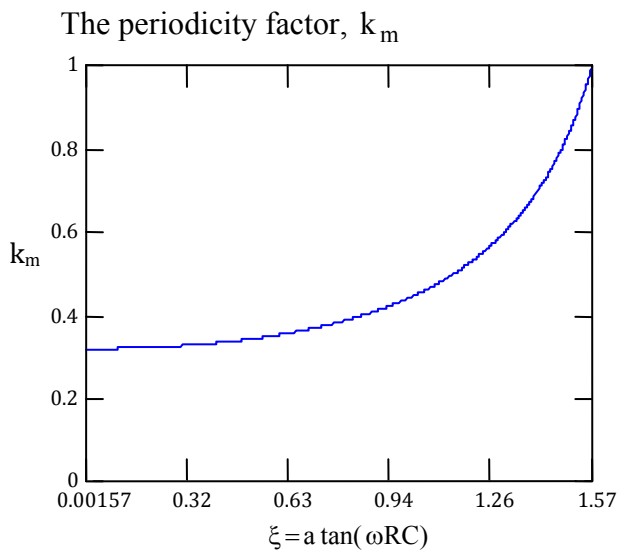


Fig.2  $k_m$  factor variation as function of load,  $\xi = \text{arctg}(\omega RC)$

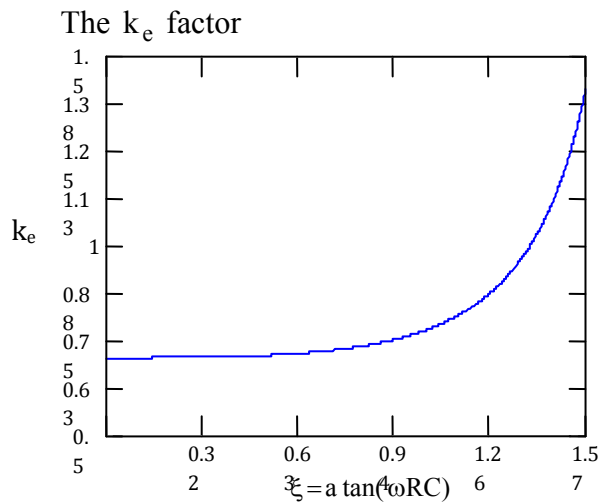


Fig.3.  $k_e$  factor variation as function of load,  $\xi = \text{arctg}(\omega RC)$

The shape factor,  $k_f$

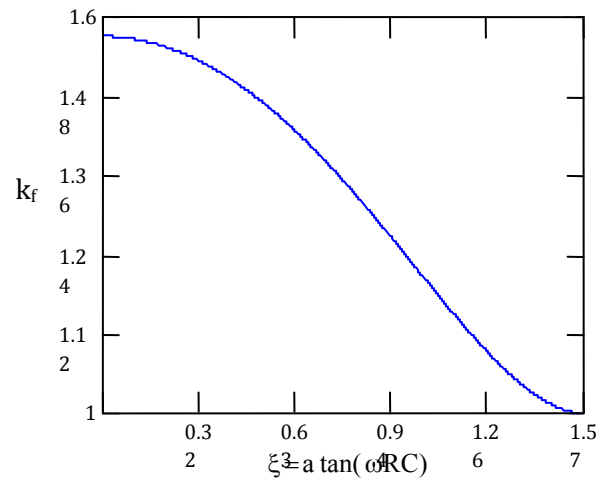


Fig.4  $k_f$  factor variation as function of load,  $\xi = \text{arctg}(\omega RC)$

The wave factor,  $k_u$

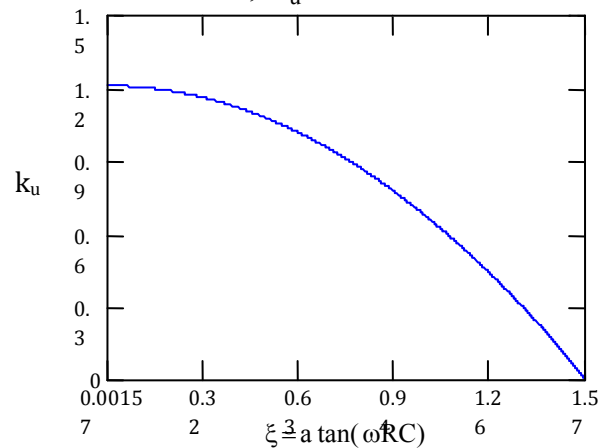


Fig.5  $k_u$  factor variation as function of load,  $\xi = \text{arctg}(\omega RC)$

### 3. CONCLUSIONS & ACKNOWLEDGMENT

Considering the limit cases, the following values of the rectifier parameters are computed:

For the pure resistive load,

$$\omega RC = 0,$$

$$\alpha_p = 0^0$$

it results:

$$k_m = 0.318 = \frac{1}{\pi}$$

$$k_e = 0.707 = \frac{1}{\sqrt{2}} \quad (17)$$

$$k_f = 1.571$$

$$k_u = 1.21$$

For pure capacitive load,

$$\omega RC = \infty,$$

$$\alpha_p \approx 90^0,$$

it results:

$$k_m = 1$$

$$k_e = \sqrt{2} \quad (18)$$

$$k_f = 1$$

$$k_u \rightarrow \infty$$

The following conclusions are deriving from the study of the half-wave voltage rectifiers equipped with uncontrolled electrical valves:

The diode turn-on angle  $\alpha_p$  has an increasing variation of load (R);

The diode turn-off angle  $\beta$  has an decreasing variation of load (R);

The periodicity factor  $k_m$  is an increasing function of load (R);

The  $k_e$  factor is an increasing function of load (R) too;

The shape factor  $k_f$  is a decreasing function of load (R);

The wave factor  $k_u$  is also a decreasing function of load (R).

### REFERENCES

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