DETERMINATION OF MAXIMUM ANGLE SLOPE THROUGH NORMAL REACTIONS TO THE WHEELS, IN STATIC AND DYNAMIC REGIME

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Abstract: *Crossing ability is the quality of the vehicle to travel on roads or in off road terrain and to overcome obstacles. Special purpose vehicles and the 4x4 require much improved performance, especially on climbing slopes. The paper aims to provide a method for determining the maximum slope angle that can climb a vehicle with two axel, starting from normal reactions to its wheels, in static and dynamic regime.*

Keywords: *vehicle dynamics, maximal road slope, normal static reaction, normal dynamic reaction.*

1. INTRODUCTION

Establishing the forces, reaction and moments on the vehicle axle are topics treated in various forms in [1-6]. Particular aspects for determining carrying capacity, maximum slope that can be climbed are treated in [1, 5, 6].

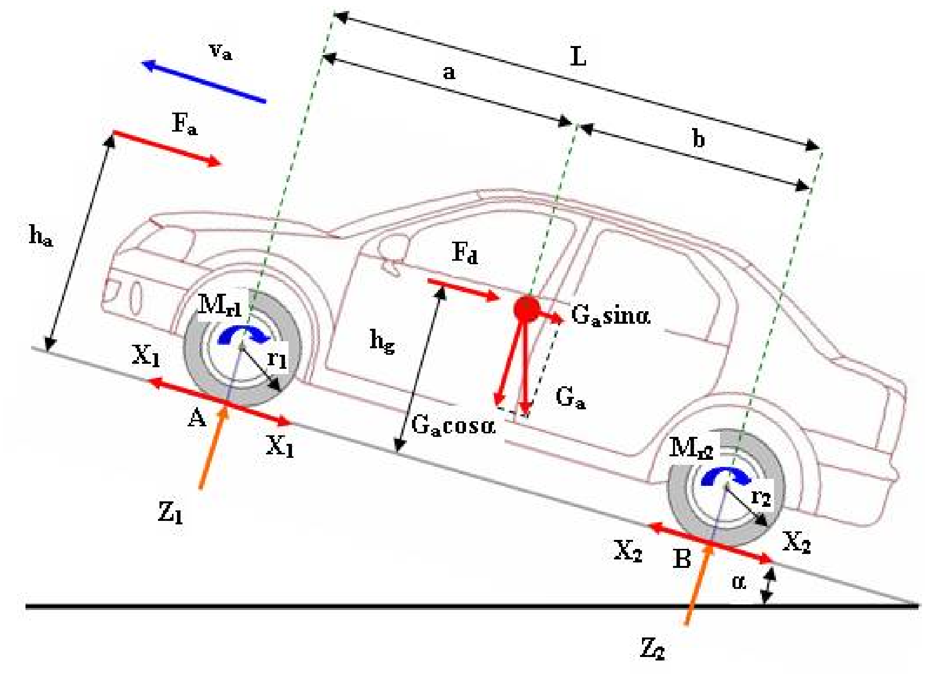

FIG. 1. Forces, reaction and moments on the vehicle with two axle

Forces, moments and reactions acting on the vehicle are shown in Fig. 1; the following notations have been made: G - weight of the vehicle; Fa - air resistance; Fd - resistance to acceleration; X1 and X2 - tangential reactions to the front and rear axle; MR1 and MR2 - rolling resistance torque for the front and rear axle; a and b - longitudinal coordinates of the center of mass; hg - height of the center of mass; ha - height of the center of pressure. Under stationary vehicle conditions, the static reactions, considered as normal to the two axles, may be determined from the equations of moments in relation to the points of contact between the wheel and the road surface, A and B. In this case we have Fa = 0; Fd = 0; MR1 and MR2 = 0 = 0. By writing the equation of moments in relation to the rear axle, the point B, the following relation obtained:

$$Z_1 \cdot L = G_a \cdot \cos(\alpha) \cdot b - G_a \cdot \sin(\alpha) \cdot h_g \tag{1}$$

where having the weight G_a as common factor the following relation is obtained:

$$Z_1 = G_a \frac{b \cdot \cos(\alpha) - h_g \cdot \sin(\alpha)}{L} \tag{2}$$

If we write the equation of moments in relation to the front axle, the point A, the following relation is obtained for the reaction on the rear axle:

$$Z_2 \cdot L = G_a \cdot \cos(\alpha) \cdot a + G_a \cdot \sin(\alpha) \cdot h_g \tag{3}$$

and by grouping the terms:

$$Z_2 = G_a \frac{a \cdot \cos(\alpha) + h_g \cdot \sin(\alpha)}{L} \tag{4}$$

Relations 2 and 4 depend on the angle of the slope where we have the vehicle and they do not depend on the coefficient of adhesion (the vehicle is stationary). From the graph in Fig. 2 it is noticed that the front axle reaction has a decreasing trend along with increasing angle α, while the rear axle loads for the same variation of the slope angle.

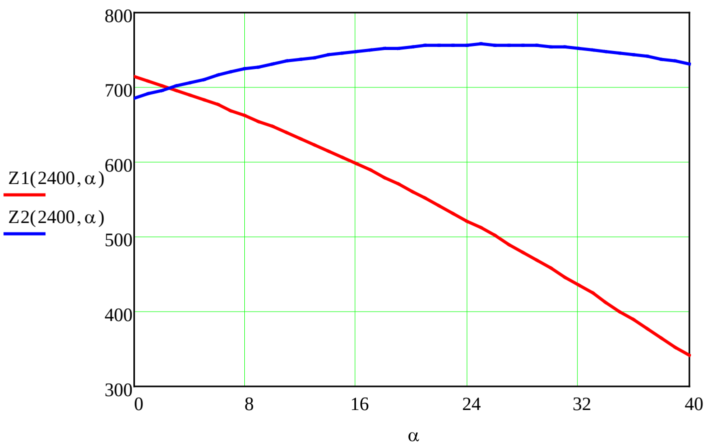

FIG. 2. Variation of vertical static reactions in case of the two axle vehicle with front axle traction, depending on the slope angle

2. VEHICLE WITH TWO AXLES MOVING ON SLOPE

In order to determine the normal dynamic reactions to the road surface, we considered the general case of a vehicle running up a ramp with trim α, in uniformly accelerated motion.

While the vehicle is running, the normal reactions to the front and rear axles change according to the type of motion. We start from the general case of a vehicle with traction or braking system applied on both axles; we then detail particular cases considering the motion and the number and position of axles. Tangential reactions X_1 and X_2 are oriented depending on the axle position: traction or braking.

As calculus assumptions for the determination of normal reaction to the two axels we consider that the vehicle is a rigid body, without taking into account the additional movement that occurs due to suspension oscillations. In order to determine the normal reaction Z_{1d} and Z_{2d} will write equations of moments in relation to the points of contact between the wheel and the road, i.e. B and A respectively, considering all the forces and moments acting on the vehicle.

By writing the moment equation in relation to point B, we obtain:

$$Z_{1d} \cdot L + M_{r1} + M_{r2} + G_a \cdot \sin(\alpha) \cdot h_g + F_d \cdot h_g + F_a \cdot h_a - G_a \cdot \cos(\alpha) \cdot b = 0 \tag{5}$$

The sum of forces perpendicular to the road surface is given by the relation:

$$Z_{1d} + Z_{2d} = G_a \cdot \cos(\alpha) \tag{6}$$

Viewing that $r_1 = r_2 = r$ şi $f_1 = f_2 = f$, we have:

$$M_{r1} + M_{r2} = f_1 \cdot r_1 \cdot Z_{1d} + f_2 \cdot r_2 \cdot Z_{2d} = f \cdot r \cdot G_a \cdot \cos(\alpha) \tag{7}$$

Where r_1 and r_2 - are the working radius of the wheels on the front and rear axle; f_1 and f_2 - rolling resistance coefficients for the wheels on the front and rear axle. Considering the relation 7, the equation 5 will be:

$$Z_{1d} \cdot L + f \cdot r \cdot G_a \cdot \cos(\alpha) + (G_a \cdot \sin(\alpha) + F_d) \cdot h_g + F_a \cdot h_a - G_a \cdot \cos(\alpha) \cdot b = 0 \tag{8}$$

from where:

$$Z_{1d} = \frac{G_a \cdot \cos(\alpha) \cdot (b - f \cdot r)}{L} - \frac{(G_a \cdot \sin(\alpha) + F_d) \cdot h_g + F_a \cdot h_a}{L} \tag{9}$$

In order to determine the normal reaction to the rear axle, Z_{2d} we take an analogue approach and we write the equation of moments in relation to point A.

$$Z_{2d} \cdot L - M_{r1} - M_{r2} - G_a \cdot \sin(\alpha) \cdot h_g - F_d \cdot h_g - F_a \cdot h_a - G_a \cdot \cos(\alpha) \cdot a = 0 \tag{10}$$

By substituting the expression 7 in the relation 10 we obtain:

$$Z_{2d} = \frac{G_a \cdot \cos(\alpha) \cdot (a + f \cdot r)}{L} + \frac{(G_a \cdot \sin(\alpha) + F_d) \cdot h_g + F_a \cdot h_a}{L} \tag{11}$$

The normal reactions from relations of Z_{1d} and Z_{2d} reveal that they depend on:
• type of vehicle movement;
• road characteristics;
• construction characteristics of the vehicle.

These relations are valid regardless the direction and the value of X_1 and X_2 tangential reaction forces and their distribution on axles.

Since tangential reactions are limited by adhesion, normal reaction to axles cannot exceed certain limits. From equilibrium of forces in the direction of the vehicle traveling speed, we obtain the relation:

$$\pm X_1 \pm X_2 = G_a \cdot \sin(\alpha) + F_a + F_d \tag{12}$$

giving:

$$G_a \cdot \sin(\alpha) + F_d = (\pm X_1 \pm X_2) - F_a \tag{13}$$

Taking into account the relation 13, the expressions 9 and 11 of normal reaction Z_1 and Z_2 will be written as:

$$Z_{1d} = \frac{G_a \cdot \cos(\alpha) \cdot (b - f \cdot r)}{L} - \frac{[(\pm X_1 \pm X_2) - F_a] \cdot h_g + F_a \cdot h_a}{L} \tag{14}$$

and, respectively:

$$Z_{2d} = \frac{G_a \cdot \cos(\alpha) \cdot (a + f \cdot r)}{L} + \frac{[(\pm X_1 \pm X_2) - F_a] \cdot h_g + F_a \cdot h_a}{L} \tag{15}$$

Relations 14 and 15 will be further particularized for a vehicle with front axle traction.

3. TWO AXLE VEHICLE WITH FRONT AXLE TRACTION

In this case in relations 14 and 15 the following replacements are made in order to determine $Z_{1dtf.}$

$$\begin{cases} X_1 = \varphi \cdot Z_{1d} \\ X_2 = -f \cdot Z_{2d} = -f \cdot G_a \cdot \cos(\alpha) + f \cdot Z_{1d} \end{cases} \tag{16}$$

and to determine Z_{2dtf} respectively:

$$\begin{cases} X_1 = \varphi \cdot Z_{1d} = \varphi \cdot G_a \cdot \cos(\alpha) - \varphi \cdot Z_{2d} \\ X_2 = -f \cdot Z_{2d} \end{cases} \tag{17}$$

giving

$$Z_{1dtf} = \frac{G_a \cdot \cos(\alpha) \cdot \lfloor b + f \cdot (h_g - r) \rfloor}{L + h_g \cdot (\varphi + f)} - \frac{F_a \cdot (h_a - h_g)}{L + h_g \cdot (\varphi + f)} \tag{18}$$

$$Z_{2dtf} = \frac{G_a \cdot \cos(\alpha) \cdot (a + f \cdot r + \varphi \cdot h_g))}{L + h_g \cdot (\varphi + f)} + \frac{F_a \cdot (h_a - h_g)}{L + h_g \cdot (\varphi + f)} \tag{19}$$

We can estimate that the height of the center of mass hg is equal to the height of the center of pressure ha and the force of air resistance F_a is lower at lower speeds. Thus, under conditions of maximum traction, the second term from the numerator of relations 18 and 19 may be neglected. The dynamic reactions depend on the road slope and the coefficient of adhesion φ.

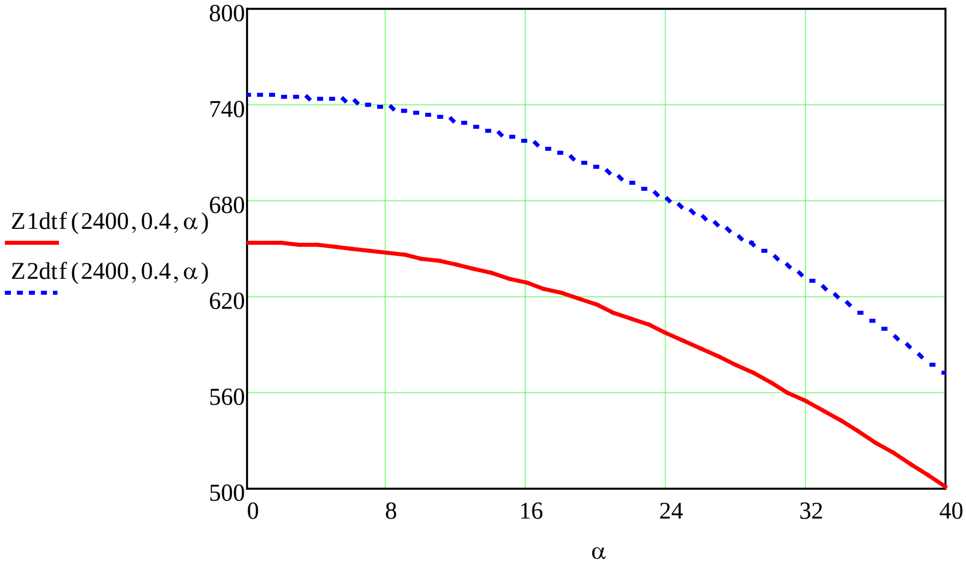

FIG. 3. Variation of vertical dynamic reactions in case of the two axle vehicle with front axle traction, depending on the slope angle

From relations 18 and 19 we notice a decreasing trend of vehicles normal reactions to axles as the slope angle α increases, Fig. 3.

Provided the same graph presents the dynamic and static vertical reactions on an inclined road, Fig. 4. shows that for a particular adhesion coefficient and a particular slope angle the static and dynamic reactions intersect.

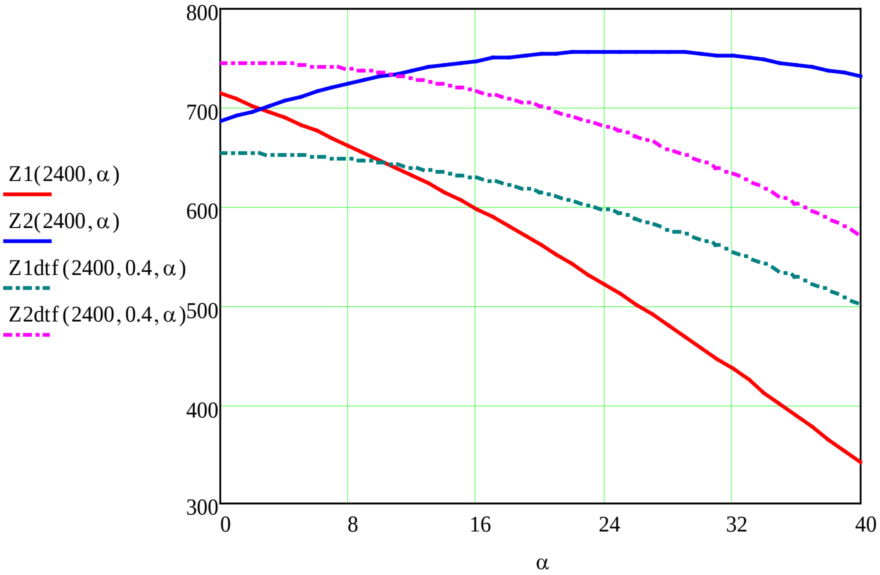

FIG. 4. Variation of vertical dynamic and static reactions in case of the two axle vehicle with front axle traction, depending on the slope angle

This means that for an angle α the static reactions are equal to those generated in dynamic conditions. Furthermore, for the reactions to both front and rear axle the intersection angle is the same. We may conclude that when the relation between normal dynamic reactions and static normal reactions is equal to 1, $Z_{1dtf}/Z_1=Z_{2dtf}/Z_2=1$, we obtain the maximum slope angle at which a vehicle may run up, as seen in figure 6.4. If $m_1= Z_{1d}/Z_1$ and $m_2= Z_{2d}/Z_2$, the values for m1 and m2, illustrated on the graph on the right of the maximum slope angle value cannot be obtained as this would result in $m_1>1$ and $m_2<1$, which is impossible to obtain in case of any type of vehicle, as seen in Fig. 5.

Figure 4 Variation of vertical dynamic and static reactions in case of the two axle vehicle with front axle traction, depending on the slope angle.

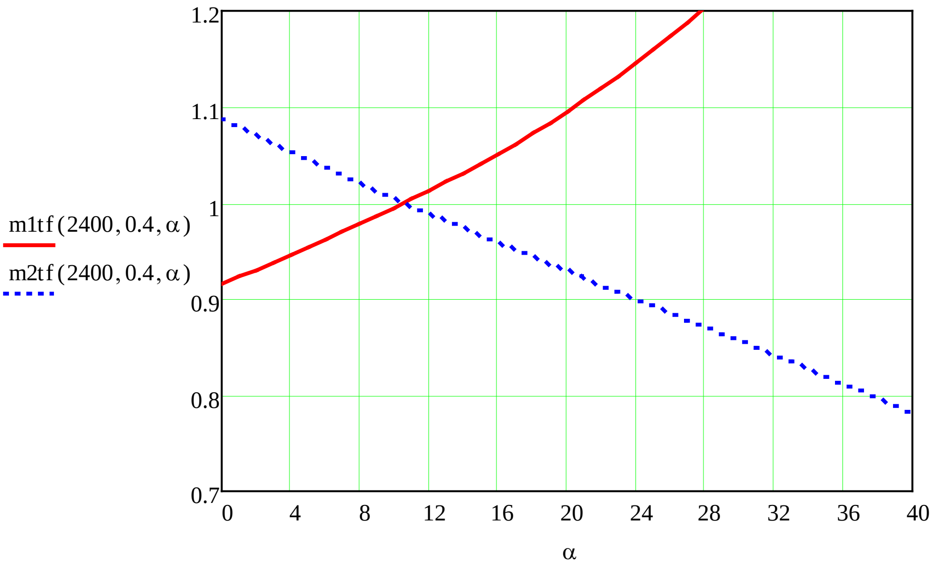

FIG. 5. Variation of dynamic change coeficient of reactions to axles in case of a vehicle with front axle traction depending on the slope angle

4. COMPARISON OF METHODS USED TO DETERMINE THE MAXIMUM SLOPE

The classical method to determine the maximum slope that a vehicle may climb, considering that the vehicle cannot roll and without taking into account the rolling resistance, uses the following formulae for 4x2 vehicles under different traction systems.

Thus, for a vehicle with front axle traction, the following formula is used:

$$\alpha_{max} = a\tan\left(\frac{b}{\frac{L}{\varphi} + h_g}\right) \tag{20}$$

Knowing the dynamic and static reactions, the maximum slope may be determined by solving the equation described by the equality $Z_{1d}/Z_1=1$ by defining two functions h(α), g(φ,α) and h(α) = g(φ,α):

$$\begin{aligned} g(\varphi,\alpha) &= \frac{\lfloor b + f\cdot(h_g - r)\rfloor\cdot G_a\cdot\cos(\alpha)}{L + (\varphi + f)\cdot h_g} \\ h(\alpha) &= \frac{G_a\cdot b\cdot\cos(\alpha) - G_a\cdot h_g\cdot\sin(\alpha)}{L} \end{aligned} \tag{21}$$

and by applying the root function that may be used to determine the maximum slope angle at which a vehicle may run up depending on the parameter φ.

The root function can only solve one equation in one unknown:

$$Rampa\max 1(\varphi,\alpha) = root(h(\alpha) - g(\varphi,\alpha),\alpha) \tag{22}$$

By comparing the result we actually notice an overlap of the values of the maximum slope angle at which a vehicle may climb under different adhesion conditions.

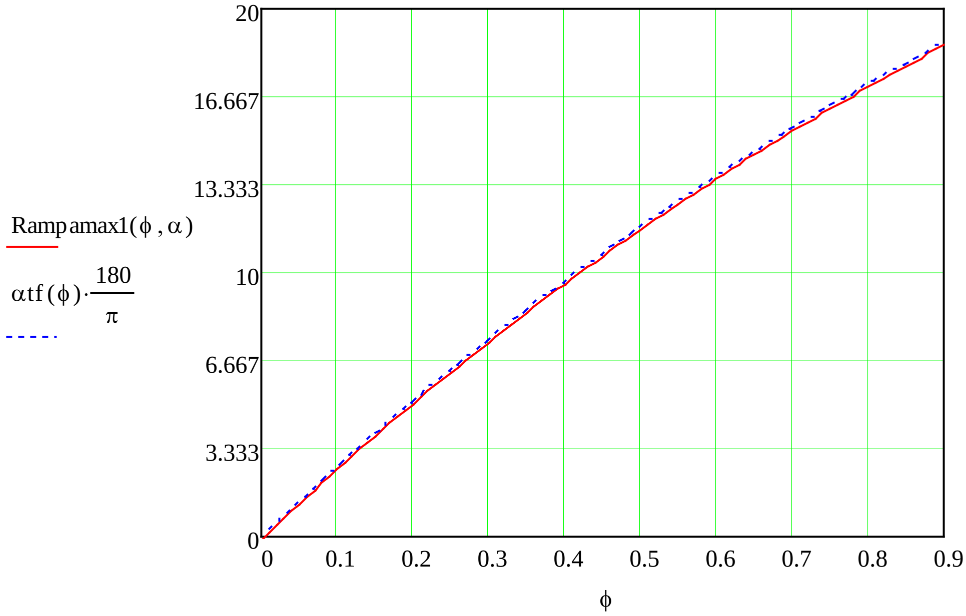

FIG. 6. Comparison of the maximum slope angle at which a vehicle may climb

CONCLUSIONS

Determination of maximum slope angle that can be climbed to a vehicle with two axle from normal static and dynamic reactions to the axle, offers similar results with other methods, for all possible traction configurations 4x2 and 4x4.

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