ROBUST LQG CONTROLLER DESIGN FOR THE SMALL UNMANNED AERIAL VEHICLE

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Abstract: *This paper deals with the dynamic optimal control of unmanned aerial vehicles (UAVs) in the presence of the stochastic plant disturbances and sensor noises. The LQR static optimal control technique of the dynamical system is used for an ideal dynamics of the aircraft, determined under conditions of no external or internal disturbance, which can be rarely met in the practice. The more realistic dynamical model applied for the controller synthesis is the random dynamical model allowing consideration of both plant disturbances and sensor noises simultaneously. The purpose of the research is to propose and implement the LQG design method for the fixed-wing UAV controller synthesis procedure.*

Keywords: *UAV, modelling, automatic flight control system, dynamic optimal control, LQG design.*

1. INTRODUCTION

Recently, the unmanned aerial aircraft (UAV) has gained a widened range of possible applications. They can be used both for military and non-military applications. The versatile applications inside the two main classes listed above predict a need for the automation of different flight phases. Automation itself serves the need to meet the necessary flight safety level, or serves as the easy-to-use onboard platform supporting the actual user of the UAV.

The recently developed and used drone taxi concept is one of the promising fields of civil application of the UAV. However, the urban area UAV applications, which are not dependent of the application features, require robust control systems ensuring both stability and dynamic performances. Robustness must be provided by the controller designed for the given UAV.

Secondly, the existing UAV regulations often require the onboard autopilot supporting UAV operator for the successful execution of the flight mission. The reason behind this research is to design an optimal, robust dynamic LQG controller for the small UAV, and to present a numerical example, using the results gained during computer aided design and analysis.

2. LITERATURE REVIEW

The problems related to the design of the optimal control systems are in the focus of attention of many scholars since many decades. The early stage, called solution of the minimum energy problem, was exhaustively examined.

The LQR and LQE problem solutions have a long history with many powerful and attractive solutions. Industrial robust control systems are duly demonstrated in [1, 3, 4, 5].

The optimal control theory is elaborated in works of [2, 5, 6, 7, 8, 9, 10, 13]. The optimal control theory is widely applied to design both LQR and LQG controllers for the UAV applications. In [11, 12, 14, 17, 19], the dynamic controller synthesis based on LQ-techniques is presented. The design criteria applied in this article leans on [15, 16].

The robust controller design will be supported by MATLAB and by its required toolboxes [20, 21]. The UAV innovative solutions are thoroughly examined in [22]. The UAV launch is always a challenging task. There is a new concept of the electromagnetic launch, outlined in [23], and the calculations related to magnetic field issues are properly outlined.

3. THE MAIN IDEA OF THE LQG OPTIMAL DYNAMICAL CONTROL

The Linear Quadratic Gaussian (LQG) control problem is formulated for the linearized, time invariant plant model that is disturbed both with process and measurements noises. The random multivariable system is given with the state and output equations as follows below [1, 3, 4, 13]:

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u} + \mathbf{G}\boldsymbol{w}; \boldsymbol{y} = \mathbf{C}\boldsymbol{x} + \boldsymbol{v} \qquad (1)$$

where **x** is a column state vector of length *n*, **u** is the control input vector of length *r*, **A** is an ($n \times n$) square state matrix; **B** is an ($n \times r$) input matrix; **y** is a column output vector; **C** is an ($m \times n$) the output matrix; $\boldsymbol{w}$(t) is the vector of the process disturbances, $\boldsymbol{\Gamma}$ is the process disturbance input matrix, and finally, $\boldsymbol{v}$ is the measurement noise.

The plant noise $\boldsymbol{w}$(t) and measurement $\boldsymbol{v}(t)$ noise are usually assumed to be the uncorrelated Gaussian random processes with zero mean values, i.e.:

$$E\{\boldsymbol{w}(t)\} = 0; \quad E\{\boldsymbol{v}(t)\} = 0; \quad E\{\boldsymbol{w}(t)\boldsymbol{v}^T(t+\tau)\} = 0 \qquad (2)$$

The covariance matrices of the two random signals are as follows below:

$$E\{\boldsymbol{w}(t)\boldsymbol{w}^T(t)\} = Q_0 > 0; \quad E\{\boldsymbol{v}(t)\boldsymbol{v}^T(t)\} = R_0 \geq 0 \qquad (3)$$

The LQG controller design procedure is based on the minimization of the well-known quadratic optimization criterion as follows below [13]:

$$J_{LQG} = \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T (\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u})dt\right\} \to Min \qquad (4)$$

where matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ represent weighting matrices of the LQR problem, such that $\boldsymbol{Q} = \boldsymbol{Q}^T \geq 0; \boldsymbol{R} = \boldsymbol{R}^T > 0$ and, $E\{.\}$ is the expectation operator. Using separation principle, the LQG design problem can be solved in two decoupled stages [3, 13]:

– Determine the Kalman-estimator optimal static gain $\boldsymbol{L}$ allowing to reconstitute the estimated $\hat{\boldsymbol{x}}$ of the state vector $\boldsymbol{x}$ (Linear Quadratic Estimator (LQE) problem);

– Calculate the optimal control law of $\boldsymbol{u} = -\boldsymbol{K}\hat{\boldsymbol{x}}$, where $\boldsymbol{K}$ is an optimal static feedback gain matrix calculated in solution of the LQR (Linear Quadratic Regulator).

Using the principle of separation outlined above, the state space representation of the observer-based controller is given as follows:

$$\hat{\boldsymbol{y}} = \boldsymbol{C}\hat{\boldsymbol{x}} \quad \} \qquad (5)$$

where Kalman-filter static gain $\boldsymbol{L}$ of the optimal observer is as follows [1, 3, 4, 5, 10, 13]:

$$\boldsymbol{L} = \boldsymbol{\Sigma}\mathbf{C}^T\mathbf{R}_0^{-1} \qquad (6)$$

Matrix $\boldsymbol{L}$ is a solution of the observer's matrix Algebraic Ricatti equation (MARE) as it is expressed below [1, 3, 4, 5, 10, 13]:

$$\boldsymbol{A}\boldsymbol{\Sigma} + \boldsymbol{\Sigma}\boldsymbol{A}^T - \boldsymbol{\Sigma}\boldsymbol{C}^T\boldsymbol{R}_0^{-1}\boldsymbol{C}\boldsymbol{\Sigma} + \boldsymbol{G}\boldsymbol{Q}_0\boldsymbol{G}^T = 0 \tag{7}$$

Using equations (1) and (5), the following state space model can be derived:

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\hat{\boldsymbol{x}}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & 0 \\ \boldsymbol{LC} & \boldsymbol{A}-\boldsymbol{LC} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \hat{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} & \boldsymbol{G} & 0 \\ \boldsymbol{B} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{w} \\ \boldsymbol{v} \end{bmatrix} \tag{8}$$

Combining equation (1) and (5), the Kalman-filter block diagram can be derived and depicted as in Fig. 1. [1, 3, 4, 5, 10, 13]:

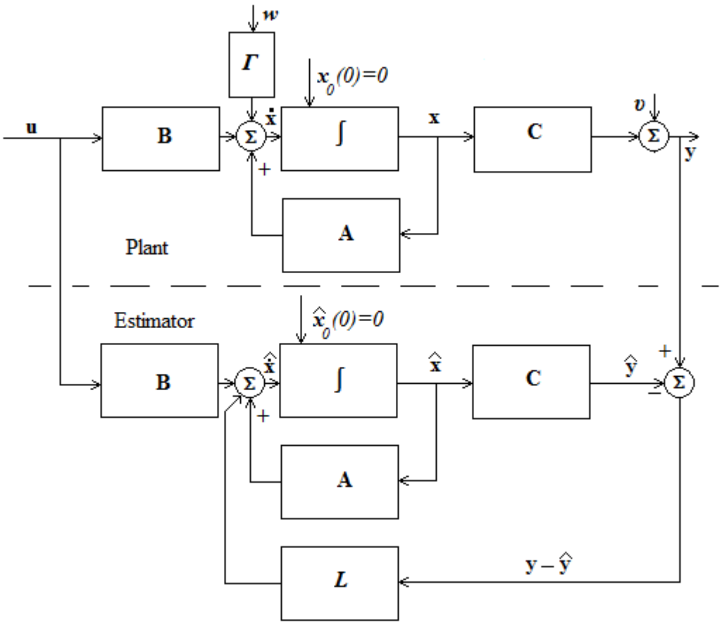


FIG. 1. Block diagram of the optimal Kalman-filter.

The Linear Quadratic Gaussian (LQG) Controller can be completed as follows (Fig. 2.):

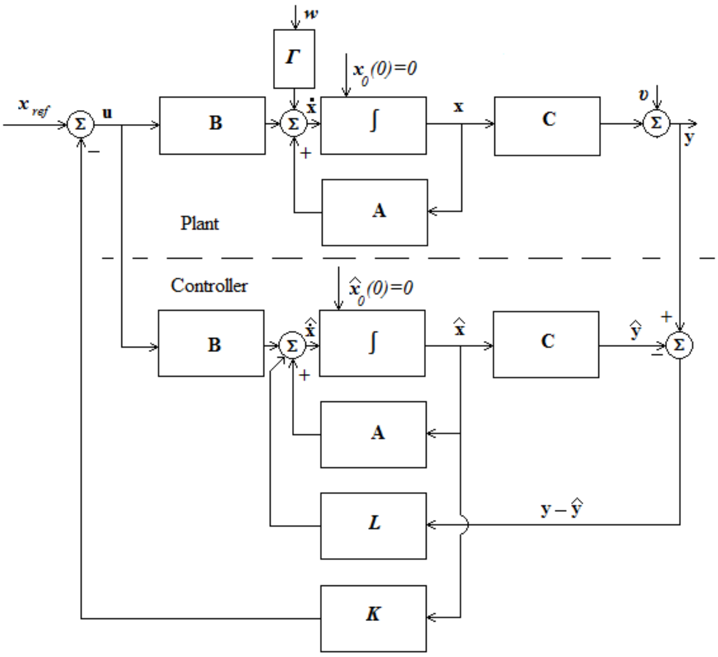


FIG. 2. Block diagram of the LQG Controller.

Combining the Kalman-filter and the LQR equations the following augmented closed loop system state equation can be derived [1, 3, 4, 5, 10, 13]:

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & -\boldsymbol{BK} \\ \boldsymbol{LC} & \boldsymbol{A}-\boldsymbol{LC}-\boldsymbol{BK} \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} & \boldsymbol{G} & 0 \\ \boldsymbol{B} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_{ref}} \\ \boldsymbol{w} \\ \boldsymbol{v} \end{bmatrix} \qquad (9)$$

4. DESIGN OF THE LQG OPTIMAL CONTROLLER FOR THE SMALL UAV

The identified dynamical model of the short period longitudinal motion of the Boomerang-60 Trainer UAV subjected to external and internal disturbances can be derived as follows [2, 18]:

$$\left.\begin{aligned} \dot{\boldsymbol{x}} = \begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \boldsymbol{Ax} + \boldsymbol{B}u + \boldsymbol{G}W = \begin{bmatrix} -0{,}9966 & 19 \\ -3{,}9794 & -12{,}991 \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} -1{,}2965 \\ -18{,}789 \end{bmatrix} [\delta_e] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} W \\ \boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{v} = [1 \quad 0] \begin{bmatrix} w \\ q \end{bmatrix} + v \end{aligned}\right\} \qquad (10)$$

where w is the vertical speed, q is the pitch rate, δ_e is the angular deflection of the elevators, W is the plant disturbance, and finally, v is the speed measurement noise.

Let us find the stabilizing LQG robust dynamic controller of the Boomerang-60 Trainer UAV depicted in Fig. 2 able to manipulate the vertical speed of the UAV. The closed loop control system of the UAV is supposed to be the oscillatory one, and the expected dynamic performances are as follows below:

$$t_s \leq 1 \; sec, \; \sigma \leq 5 \; \% \qquad (11)$$

where t_s is the settling time calculated for 2% of the tolerance field, and, σ represents the overshoot percent.

By using the $\mathbf{A}, \mathbf{B}$ pair of matrices, the system controllability has been evaluated. The controllability matrix was calculated to be [20, 21]:

$$\boldsymbol{Co} = \begin{bmatrix} -1{,}2965 & -355{,}6989 \\ -18{,}789 & 249{,}2472 \end{bmatrix} \qquad (12)$$

which has a rank of 2, i.e. the UAV dynamical system can be considered for the controllable one.

By using the $\mathbf{A}, \mathbf{C}$ pair of matrices, the observability matrix has been calculated to be [20, 21]:

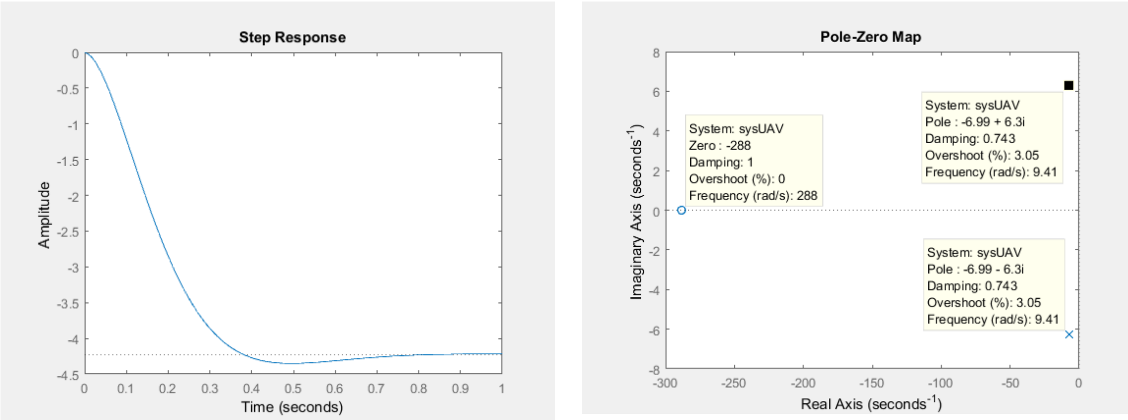
$$\boldsymbol{Ob} = \begin{bmatrix} 1 & 0 \\ -0{,}9966 & 19 \end{bmatrix} \qquad (13)$$

which has a rank of 2, i.e. the UAV dynamical system is the observable one.

The time domain behavior of the longitudinal motion of the UAV has been analyzed. The result of the computer simulation can be observed in Fig. 3.a. The input of the UAV was the unit step change in the elevator angular position, i.e. $\delta_e = 1(t) \; deg$. From Fig 3.a it can be easily determined that the UAV vertical speed oscillates around its steady-state value.

The pole-zero map of the UAV's dynamical system defined by equation (10) can be seen in Fig. 3.b. The open loop UAV has two poles on the complex plane. The UAV's behavior is determined by the pair of complex conjugate roots called dominant, and located at $s_{1,2} = -6{,}99 \pm 6{,}3 \, i$ on the right-half of the complex plane.

That pair of poles determines the UAV's time domain transient behavior with the damping ratio of $\xi = 0{,}743$, and with the overshoot percent determined by those dominant roots at 3,05% [20, 21].



(a) (b)

FIG. 3. The open loop behavior of the UAV (MATLAB-script: the author).

Using the separation principle, the LQG controller synthesis method consists of two main phases. The first one is the LQR design of the deterministic control system. The second stage is the design of the optimal Kalman-filter to estimate states. The LQR design phase was about finding the optimal state feedback gain matrix. Firstly, unit weights were applied in LQR design phase, and the UAV's closed loop control system has been tested both in time domain and in frequency domain. The system designed that way had dynamic performances different from those recommended by equation (11). After that, many trials were made to schedule dynamic performances into the ranges defined by equation (11). To ensure that the dynamic performance fits in the ranges defined before, the following weighting matrices were chosen heuristically and applied to minimize linear quadratic integral performance index:

$$\boldsymbol{Q} = \begin{bmatrix} 10000000000 & 0 \\ 0 & 0.000001 \end{bmatrix} \tag{14}$$

The optimal feedback gain matrix ***K*** and the Lyapunov (cost) matrix ***P*** were calculated using MATLAB software to be as follows [20, 21]:

$$\boldsymbol{K} = [-998{,}6082 \quad -13{,}2355]; \boldsymbol{P} = 10^6 * \begin{bmatrix} 6{,}9975 & 0{,}0486 \\ 0{,}0486 & 0{,}0037 \end{bmatrix} \tag{15}$$

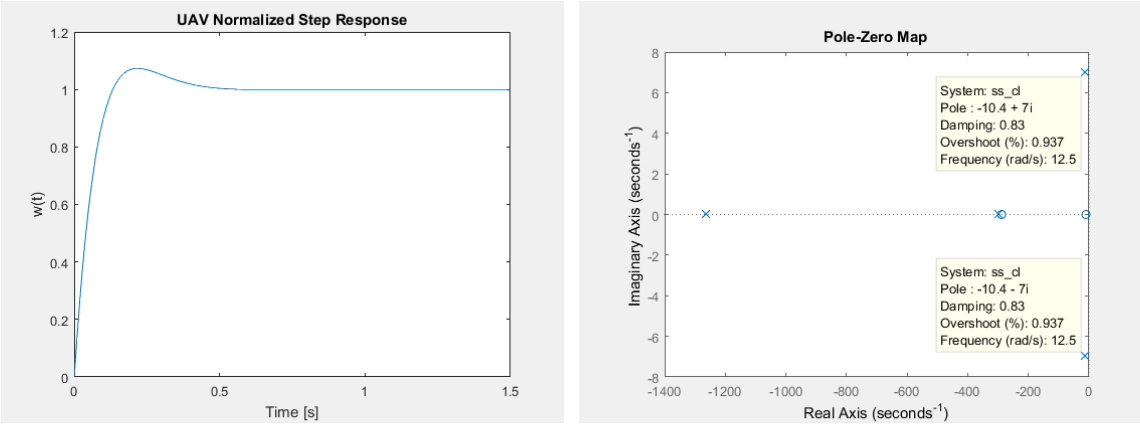
The linear quadratic estimator (LQE) optimal static gain was calculated for the following noise intensities [20, 21]:

$$Q_0 = 0{,}1; \ R=0{,}001 \tag{16}$$

The optimal Kalman-filter gain matrix ***L*** and the Lyapunov (cost) matrix **Σ** were calculated using MATLAB software to be as follows [20, 21]:

$$\boldsymbol{L^T} = [6{,}8207 \quad -1{,}0496]; \boldsymbol{\Sigma} = \begin{bmatrix} 0{,}0068 & -0{,}0010 \\ -0{,}0010 & 0{,}0003 \end{bmatrix} \tag{17}$$

The LQG regulator has been formed using MATLAB. The closed loop step response time domain behavior can be seen in Fig. 4. The UAV's closed loop control system is subjected to the input of $w_{ref} = 1(t)$.



(a) (b)

FIG. 4. The behavior of the UAV's closed loop control system (MATLAB-script: the author).

Considering Fig. 4.a, it is easy to see that the UAV's closed loop system based on an LQG robust controller responds very fast, and the unit input is maintained. The closed loop control system poles and dynamic performances are tabulated in Table 1. [20, 21]:

Table 1. Closed Loop Poles and Performances

Pole	Damping ratio ξ	Frequency (rad/sec)	Time Constant (seconds)
$-10{,}4 + 7\,i$	0.83	12,5	0,0961
$-10{,}4 - 7\,i$	0,83	12,5	0,0961
–296	1	296	0,00338
–1260	1	1260	0,000793

Fig. 4.b demonstrates that the four poles of the closed loop control system have a pair of complex conjugates and two roots which are real, negative values located at large distances from the dominant roots. In other words, it means that the UAV's closed loop control system behavior was mostly determined by the dominant roots, and the effects of the remaining two roots can be neglected.

The dynamic performances of the UAV's closed loop dynamical system based on robust LQG controller were found to be:

$$t_s = 0{,}5\ sec,\ \ \sigma = 0{,}937\ \% \tag{18}$$

Comparing those dynamic performances preliminarily defined by equation (11), and those performances provided for the UAV closed loop control system by the LQG controller based upon optimal gains of ***K*** and ***L*** given in equation (18), it can be stated that weights used for optimal design and given by equations (14) and (16), being chosen totally heuristically, properly fit the design problem.

Results of the computer simulation in frequency domain can be seen in Fig. 5. It can be concluded that the closed loop control system based on the LQG robust dynamic controller is stable, i.e. it has positive gain and positive phase margins. If stricter criteria are to be set for the gain margin, the weights defined by equation (14) applied in LQR design phase, or the LQE weights defined by equation (16) must be scheduled and varied to ensure both open loop control system and closed loop control system dynamic performances.

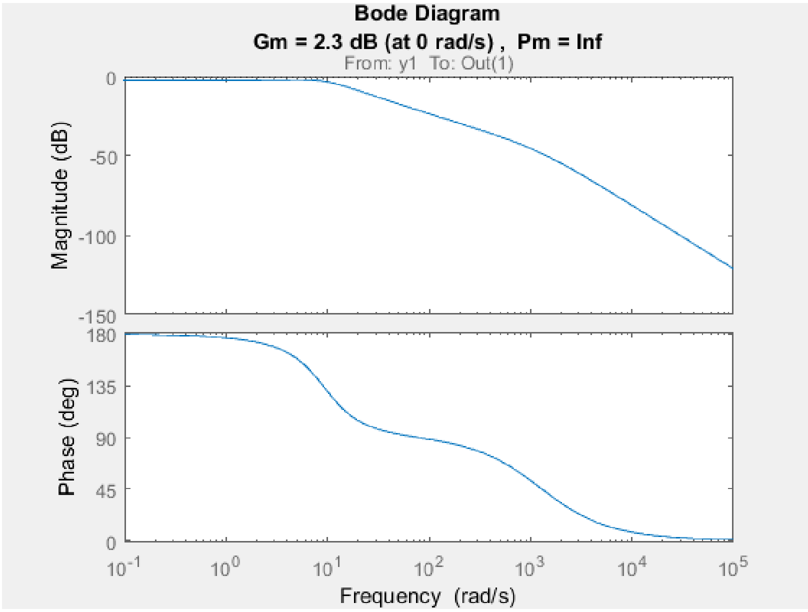


FIG. 5. The behavior of the UAV's open loop control system in the frequency domain (MATLAB-script: the author).

5. CONCLUSIONS

The main goal of this research was to solve the LQG design problem. The air in which UAVs fly is never still. The air turbulence can be considered as plant disturbance applied to the UAV's dynamical model.

The UAV on-board measurement process is a noisy one, due to several reasons. The measurement noise is modelled in output equation of the state space model of the UAV.

Due to the lack of UAV control system data, Bryson's Rule cannot be applied during the LQR design phase. Instead, the unit weighting principle was implemented. The proper set of weighting matrices applied in LQR design stage was found heuristically. The proposed weighting matrices ensured the stable behavior of the UAV's closed loop control system. Moreover, the dynamic performances advised for the small UAV were also within those tolerance domains defined to be met.

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