AN ALGORITHM FOR SYNTHESIS OF WELTI'S SIGNALS

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Abstract: *In the new generation wireless communication systems a large number of users simultaneously exploit a common communication channel. As a result the limited natural source – electromagnetic spectrum is used very effectively. This positive effect is reached on the base of ultra-wide band complex signals. With regard an algorithm for synthesis of a type of signals with complex inner structure, called Welti's signals, is suggested in the paper. It could be applied in the development of new generation communication system with code division multiple access (CDMA).*

Keywords*: Algorithm, Welti's signals, CDMA.*

1. INTRODUCTION

The wireless communication systems of third generation (3G) and beyond must provide very high rate of information transmission (up to 100 [Mb/s]) [3]. In order to satisfy this strong condition the information should be conveyed by means of ultra-wide band complex "transport" signals. In this case, in the transmitter the spectrum band F of the messages is widened by a special spreading sequences (SS) B times (B >> 1). In the receiver, the "arrived" signals are compressed in the initial bandwidth F. As a result of these manipulations, the signal-to-noise ratio (SNR) is improved B times [1,5]. With regard, the quantity B is named "processing gain" [5].

The spreading sequences (SSs) in the above described communication systems (called "ultra-vide band systems" (UWB) or "spread spectrum systems" (SSS) are used as logical addresses of the users, which exploit simultaneously the common communication channel. In order to avoid the negative effects caused by multipath spreading of the electromagnetic waves (i.e. so-named "self-interference" (SI)) and multi access interferences (MAI), the SSs must have auto-correlation function (ACF) similar to a delta-pulse and close-to-zero cross-correlation functions (CCF) for all pairs of SSs. These conditions are very strong and despite of the efforts taken during the last fifty years, only several families of SSs with desired correlation properties are found. With regard the paper aims to suggest an algorithm for synthesis of the so-named Welti's signals, which are one of the optimal families.

The paper is organized as follows. First, the basics of the Welti's signals are recalled. After that our algorithm for synthesis of generalized Welti's signals is presented. At the end some conclusions and directions for further exploration are given.

2. BASICS OF GENERALIZED WELTI'S SIGNALS

In 1961 the American theoretician Welti introduced two types of signals called D and E signals (codes) [6]. Simultaneously and independently Golay invented the so-called *complementary series* [2]. At the beginning no connection between D -signals and complementary series had been known, but after several years *Turin* and *Varakin* found that the D-signals can be viewed as a family of signals, formed recurrently from a single pair of Golay's signals (complementary series) [5]. This allowed the positive features of the E codes to be explicitly explained. In fact, E signals are generated by scrambling of two modified D-signals, which are a Golay's pair (before modification). As a result, the ACF of the E- signals is similar to a delta-pulse (i.e. the ACF has only a main lobe and is free from any side-lobes). With regard, in 1985 Luke proposed the so-named Welti's signals [4]. They are families of E –codes, possessing both a thumbtack-shape ACF and zero CCF for all two different members of a family. The classical Luke's definition can be sketched as follows.

Definition 1: Welti's signals (WSs) are a family $\{\varepsilon_k(j)\}_{j=0}^{n-1}$, k = 1, 2,…, m of signals, built from n "*elementary pulses*" (*chips*) $\varepsilon_k(j)$, k = 1, 2,…, m, j = 0, 1,…, n-1 with duration τ_0. The indices k and j show the number of the signal in the family and the position of the chip in a signal. The chips belong to the set $\{+a,-a,+b,-b\}$, which elements are orthogonal, i.e.:

$$(\pm a)\cdot(\pm b)=0,\ (+a)\cdot(+a)=1,$$
$$(+a)\cdot(-a)=1,\ (+b)\cdot(+b)=1,$$
$$(+b)\cdot(-b)=1 \qquad (1)$$

The CCF of the k-th and l-th signals of the WSs is:

$$R_{kl}(r)=\begin{cases}0, k\neq l,\\ 0, k=l\cap r\neq 0,\\ n, k=l\cap r=0.\end{cases} \qquad (2)$$

Here $r\cdot\tau_0$ is the time-shift, k = 1, 2,…, m, l = 1, 2,…, n and the CCF transforms in the ACF of the k-th signal when k = l.

From Definition 1 follows that WSs satisfy completely all correlation conditions which must meet a family of SSs exploited in a wireless communication system with code-division and multiple access (CDMA) [1,3,5]. Anyway, the classical WSs have a drawback, because they admit usage of only binary phase modulation (binary phase shift keying (BPSK)). With regard, it is necessary to introduce *generalized Welti's signals* (*GWS*s) as follows.

Definition 2: Generalized Welti's signals are a family $\{\varepsilon_k(j)\}_{j=0}^{n-1}$, k = 1, 2,…, m of signals, built from n elementary pulses (chips) $\varepsilon_k(j)$, k = 1, 2,…, m; j = 0, 1,…, n-1 with duration τ_0. The ACFs and CCFs of the signals satisfy the condition (2) and every elementary pulse belongs to the *s.p.*-set

$$\{a_1(1), a_1(2),..., a_1(p), a_2(1), a_2(2),..., a_2(p),$$
$$a_5(1), a_5(2),..., a_5(p)\},$$

containing only complex orthogonal elements:

$$a_k(u)a_l^*(v)=\begin{cases}0, k\neq l, k,l=1,2,...,s;\\ \exp[t\cdot(2\pi/p)\cdot(u-v)],\end{cases}$$
$$k=l, u,v=0,1,p-1. \qquad (3)$$

Here the symbol "*" means "complex conjugation" and $i=\sqrt{-1}$.

Physically it is not hard to realize the WSs by means of s carrier frequencies $f_1, f_2,\ldots,f_s$, which phases take p values, i.e.:

$$\varepsilon_k(l)=\begin{cases}U_m\cdot\exp\{i[\omega_k(j)\cdot t+\varphi_k(j)]\},\\ \quad j\cdot\tau_0\leq t<(j+1)\cdot\tau_0;\\ 0, t<j\cdot\tau_0\cup(j+1)\cdot\tau_0\geq t,\\ k=1,2,...,m; j=0,1,...,n-1.\end{cases} \qquad (4)$$

Here

$$U_m=1[V], \omega_k(j)\in\{2\pi f_1, 2\pi f_2,....,2\pi f_s\},$$
$$\varphi_k(j)\in\{0,(2\pi/p),...,[2\pi(p-1)/p]\}.$$

The fact that the elementary pulses are orthogonal facilitates the analysis of the features of the known WSs and GWSs, but makes significantly difficult the synthesis of new WSs or GWSs. This peculiarity is a consequence of that the orthogonal pulses are "divisors of the zero" from the algebraic point of view. More specifically, let us consider the equation:

$$X^2+(b-a)\cdot X=0. \qquad (5)$$

According to usual algebra it must have 2 roots, because its degree is 2. It is easy to calculate them X = 0, X = a – b. Surprisingly, if a and b are orthogonal as in (2) or (3), then the equation (5) "obtains" an extra root X = a.

3. AN ALGORITHM FOR SYNTHESIS OF GENERALIZED WELTI'S SIGNALS

The above example shows that powerful tools of the modern algebra can not be directly applied for synthesis of new GWSs. In fact, most valuable results in this area have been obtained after exhaustive computer researches [4]. With regard, in the rest part of the paper an algorithm for direct synthesis of GWSs will be suggested. It is based on the following proposition.

Proposition 1: Let B and C:

$$B=\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1s} \\ b_{21} & b_{22} & \dots & b_{2s} \\ \dots & \dots & \dots & \dots \\ b_{p1} & b_{p2} & \dots & b_{ps} \end{bmatrix}, \left|b_{jk}\right|=1, p\le s \quad (6)$$

$$C=\begin{bmatrix} c_{11} & c_{12} & \dots & c_{1p} \\ c_{21} & c_{22} & \dots & c_{2p} \\ \dots & \dots & \dots & \dots \\ c_{p1} & c_{p2} & \dots & c_{ps} \end{bmatrix} \quad (7)$$

be unimodular matrices (in particular they can be Hadamard matrices), i.e.:

$$BB^*=\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1s} \\ b_{21} & b_{22} & \dots & b_{2s} \\ \dots & \dots & \dots & \dots \\ b_{p1} & b_{p2} & \dots & b_{ps} \end{bmatrix}.$$

$$\cdot\begin{bmatrix} b^*_{11} & b^*_{12} & \dots & b^*_{1s} \\ b^*_{21} & b^*_{22} & \dots & b^*_{2s} \\ \dots & \dots & \dots & \dots \\ b^*_{p1} & b^*_{p2} & \dots & b^*_{ps} \end{bmatrix}=\begin{bmatrix} s & 0 & \dots & 0 \\ 0 & s & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & s \end{bmatrix} \quad (8)$$

$$CC^*=\begin{bmatrix} c_{11} & c_{12} & \dots & c_{1p} \\ c_{21} & c_{22} & \dots & c_{2p} \\ \dots & \dots & \dots & \dots \\ c_{p1} & c_{p2} & \dots & c_{pp} \end{bmatrix}.$$

$$\cdot\begin{bmatrix} c^*_{11} & c^*_{12} & \dots & c^*_{1p} \\ c^*_{21} & c^*_{22} & \dots & c^*_{2p} \\ \dots & \dots & \dots & \dots \\ c^*_{p1} & c^*_{p2} & \dots & c^*_{pp} \end{bmatrix}=\begin{bmatrix} p & 0 & \dots & 0 \\ 0 & p & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & p \end{bmatrix} \quad (9)$$

Let $D_1, D_2, \dots, D_p$ be a set of derivative matrices, obtained from the matrices B and C by multiplication of the rows of B by elements of a single row of C:

$$D=\begin{bmatrix} d_j(1,1) & d_j(1,2) & \dots & d_j(1,s) \\ d_j(2,1) & d_j(2,2) & \dots & d_j(2,s) \\ \dots & \dots & \dots & \dots \\ d_j(1p,1) & d_j(p,2) & \dots & d_j(p,s) \end{bmatrix}=$$

$$=\begin{bmatrix} c_{j1}b_{11} & c_{j1}b_{12} & \dots & c_{j11}b_{1s} \\ c_{j2}b_{21} & c_{j2}b_{22} & \dots & c_{j2}b_{2s} \\ \dots & \dots & \dots & \dots \\ c_{jp}b_{p1} & c_{jp}b_{p2} & \dots & c_{jp}b_{ps} \end{bmatrix}, j=1,2,\dots,p \quad (10)$$

Let the s-set $\{a_1(1), a_2(1), \dots, a_s(1)\}$ contains s complex orthogonal elements, satisfying (3) in the case u = v = 1. Then the sequences:

$$\begin{cases} \varepsilon_k(0)=d_k(1,1)a_1(1), \varepsilon_k(1)=d_k(1,2)a_2(1), \dots, \\ \varepsilon_k(s-1)=d_k(1,s)a_s(1) \\ \varepsilon_k(s)=d_k(2,1)a_1(1), \varepsilon_k(s+1)=d_k(2,2)a_2(1), \dots, \\ \varepsilon_k(2s-1)=d_k(2,s)a_s(1) \\ \dots\dots\dots\dots\dots\dots \\ \varepsilon_k(s(p-1))=d_k(p,1)a_1(1), \varepsilon_k(s+1)= \\ d_k(p,2)a_2(1), \dots, \varepsilon_k(sp-1)=d_k(p,s)a_s(1) \end{cases}$$

$$k=1,2,\dots,p \quad (11)$$

are a GWS.

Proof: Let us consider the CCF of the k-th and l-th sequences from the family (11):

$$R_{kl}(r)=\begin{cases} \sum_{j=0}^{sp-1-r}(\varepsilon_k(j)\cdot\varepsilon_l^*(j+r)), 0\le r\le r-1, \\ \sum_{j=0}^{sp-1-|r|}(\varepsilon_k^*(j)\cdot\varepsilon_l(j+r)), -(n-1)\le r<0. \end{cases} \quad (12)$$

Here 2 cases are possible. In order to simplify their analysis the following auxiliary indices will be used:

$$j_v=\left[\frac{j}{s}\right], j\equiv j_u (\bmod s),$$
$$r_v=\left[\frac{r}{s}\right], r\equiv r_u (\bmod s) \quad (13)$$

With regard to (13) the products in (12) can be evaluated as follows:

1) If $r_u\ne 0$ then:

$$\varepsilon_k(j)\cdot\varepsilon_l^*(j+r)=\left[d_k(j_v\cdot s+j_u)\cdot a_{j_u}(1)\right]\cdot\left[d_l(j_v\cdot s+j_u+r)\cdot a_{j_u+r_u}(1)\right]^*=0 \quad (14)$$

because $j_u+r_u\neq j_u$ and $a_{j_u}(1)\cdot a^*_{j_u+r_u}(1)=0$, according to (3).

2) If $r_u=0$ then:

$$\varepsilon_k(j)\cdot\varepsilon_l^*(j+r)=\left[d_k(j_v\cdot s+j_u)\cdot a_{j_u}(1)\right]\cdot\left[d_l(j_v\cdot s+j_u+r)\cdot a_{j_u+r_u}(1)\right]^*= \\ =d_k(j_v\cdot s+j_u)\cdot d_l^*((j_v+r_v)\cdot s+j_u) \quad (15)$$

because $j_u+r_u=j_u$ and $a_{j_u}(1)\cdot a^*_{j_u+r_u}(1)=1$, according to (3).

As a result of (15), the formula (12) can be simplified as follows:

$$R_{kl}(r)=\sum_{j_v=0}^{p-r}\sum_{j_u=0}^{s-1}d_k(j_v\cdot s+j_u)\cdot d_l^*((j_v+r_v)\cdot s+j_u) \\ =\sum_{j_v=1}^{p-r}c_{k,j_v}\cdot c^*_{l,j_v+r_v}\left[\sum_{j_u=1}^{s}b_{j_v,j_u}b^*_{j_v+r_v,j_u}\right]. \quad (16)$$

Here:

$$\sum_{j_u=1}^{s}b_{j_vj_u}b^*_{j_v+r_v,j_u}=\begin{cases}0, r_v\neq 0;\\ s, r_s=0\end{cases} \quad (17)$$

according to (8). But:

$$\sum_{j_v=1}^{p}c_{k,j_v}\cdot c^*_{l,j_v}=\begin{cases}0, k\neq l;\\ p, k=l\end{cases} \quad (18)$$

with regard to (9).

From (14), (16), (17) and (19) follows that the CCF of the k-th and l-th sequences from the family (11) satisfy the condition (2) with $n=p\cdot s$. Now it should be seen that elements of the sequences (11) are orthogonal. This observation ends the proof of the Proposition 1. Taking into account the Proposition 1, the following Algorithm for synthesis of GWelti's signals can be substantiated.

Algorithm for synthesis of Generalized Welti's signals

1) Find the optimal values of s and p so that the given frequency band $\Delta F=B\cdot F$ of the wireless communication system to be "packed" uniformly by electromagnetic spectrums of the transport signals. Here the following limitations should be kept:

1a) If a p-phase modulation (p-phase shift keying (p-PSK) will be used in the communication system, then the frequency gap Δf among carrier frequencies $f_1, f_2, ..., f_s$ must be:

$$\Delta f=\frac{2}{p\cdot\tau_0}. \quad (19)$$

This provides avoiding of the folding of the spectrums of the modulated signals and guarantees their exact separation by rejective frequency filters.

1b) The carrier frequencies $f_1, f_2, ..., f_s$ must be chosen according to the rules:

$$s=\frac{\Delta F}{\Delta f}, \quad (20)$$

$$f_1=f_0+(l-1)\cdot\Delta f, l=1,2,...,s. \quad (21)$$

2) Take a set of orthogonal elements:

$$a_k(1)=\begin{cases}\exp[i\cdot\omega_k(1)\cdot t], 0\leq t<\tau_0;\\ 0, \tau_0<0\cup\tau_0\geq t, k=1,2,...s.\end{cases} \quad (22)$$

Here $\omega_1(1), \omega_2(1), ..., \omega_s(1)$ is an arbitrary permutation of the angle frequencies $2\pi f_1, 2\pi f_2, ..., 2\pi f_s$.

3) Find unimodular matrices B and C, satisfying conditions (8) and (9) respectively and form the p-set of matrices $D_j, j=1,2,...,p$ according to (10).

It should be seen that:

$$p=s \quad (23)$$

is the optimal value of p, because this is the maximal possible number of users, exploiting simultaneously the system frequency bandwidth ΔF without SI and MAI, and $p\leq s$ (see (6)). Moreover, in this case always exist matrices B satisfying (8). Their general form is:

$$B=\begin{bmatrix}g^{0.0} & g^{0.1} & ... & g^{0.(s-1)}\\ g^{1.0} & g^{1.1} & ... & g^{1.(s-1)}\\ ... & ... & ... & ...\\ g^{(s-1).0} & g^{(s-1).1} & ... & g^{(s-1).(s-1)}\end{bmatrix} \quad (24)$$

Here g is an arbitrary s-th primitive root of unity:

$$g = \exp\left(i\frac{2\pi\omega}{s} \right) \quad (25)$$

w and s are coprime.

The matrix C can be created by (24) also.

4) Form a GWS $\{\varepsilon_k(j)\}_{j=0}^{n-1}$, k = 1, 2,…, m according to (11) and accounting that:

$$m = s \quad (26)$$

If the matrices B and C are created by (24), then the elementary pulses (chips) can be presented in the form:

$$\varepsilon_k(j) = \begin{cases} U_m \cdot \exp\{i[\omega_k(j) \cdot t + \varphi_k(j)]\}, \\ j \cdot \tau_0 \le t < (j+1) \cdot \tau_0; \\ 0, t < j \cdot \tau_0 \cup (j+1) \cdot \tau_0 \ge t, \\ k = 1,2,...,s; j = 0,1,...,s^2 - 1. \end{cases} \quad (27)$$

where:
$U_m = 1[V]$; $\varphi_k(j) \in \{0, (2\pi/2), ..., [2\pi(s-1)/s]\}$.

4. CONCLUSIONS

In the paper an algorithm for synthesis of families of ultra wide band radio signals, named Generalized Welti's signals, is suggested. Their application in the modern CDMA wireless communication systems provides avoiding both the negative effects caused by multipath spreading of the electromagnetic waves and multi access interferences. The valuable positive features of the GWSs are a consequence of their complex inner structure, reached by a simultaneous application of frequency and phase modulations, forming ACFs similar to a delta-pulse and close-to-zero CCFs for all pairs of a family.

In a general situation the synthesis of the GWSs (i.e. the mathematical research of adequate rules for frequency and phase modulations) is very difficult due to the presence of the so-named zero divisors. From this fact follows the importance of the suggested in the paper algorithm, which is applicable for arbitrary types of discrete phase and frequency modulations. Moreover, it is possible to extend the maximal possible number of users, exploiting simultaneously the system frequency bandwidth ΔF without SI and MAI, from p to s.p. This can be reached by applying of "a super GWS family", comprising s GWSs, formed by means of different permutations $\omega_1(1), \omega_2(1), ..., \omega_s(1)$ of the angle frequencies $2\pi f_1, 2\pi f_2, ..., 2\pi f_s$. This opportunity will be explored in more details in a future work.

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