TRANSMISSION LINES ANALYSIS USING CASCADES T FILTERS

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Abstract: *This paper proposes a method of determining the characteristic impedance and the equation of propagation in transmission lines, by equivalence of the line segment with an elementary T filter. We obtain thus the equivalence between a cascaded T filters network and a transmission line arbitrarily chosen. The mathematical model used is based on string theory.*

Keywords: *transmission line, T filter, characteristic impedance, wave propagation.*

1. INTRODUCTION

A transmission line segment can be represented - considering its functionality, as an infinity of elementary T filters networks connected in series (fig. 1), forming a cascaded network with Z_e equivalent impedance (Morariu *et al*., 2009:23).

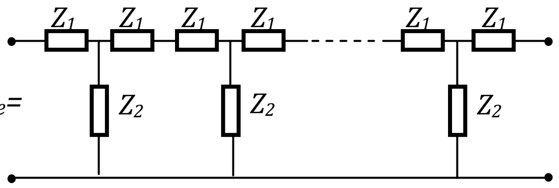

Fig. 1. The network of cascaded T filters

In the sense of the consideration cited, the transmission line can be rated as having equivalent structure from Figure 2.

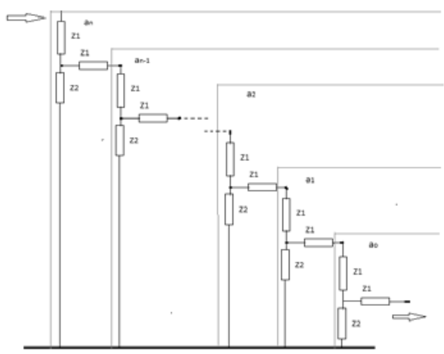

Fig. 2 The equivalent structure of the transmission line

2. CHARACTERISTIC IMPEDANCE OF THE TRANSMISSION LINE

Impedance in different points of the structure is determined to be equivalent:

$$a_0 = Z_1 + Z_2 \quad (1)$$

$$a_1 = Z_1 + Z_2 \| (Z_1 + a_0) \quad (2)$$

$$a_1 = Z_1 + Z_2 \| (Z_1 + a_1) \quad (3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_1 = Z_1 + Z_2 \| (Z_1 + a_{n-1}) \quad (4)$$

where a_n is a recursive string.

If $n \rightarrow \infty$, it follows that $\lim a_n = \lim a_{n-1}$ and $\lim a_n = l$. So,

$$l = Z_1 + Z_2 \| (Z_1 + l) \quad (5)$$

$$l = Z_1 + \frac{Z_2 (Z_1 + l)}{Z_2 + Z_1 + l} \quad (6)$$

$$l^2 = Z_1^2 + 2Z_1 Z_2 \quad (7)$$

$$l = \sqrt{Z_1^2 + 2Z_1 Z_2} \quad (8)$$

The elementary network (Morariu *et al*., 2009:23) consists of components R, L, G and C (fig. 3).

Substituting in (8) equivalent parameters of fig. 3 results:

$$l = \sqrt{Z_1^2 + 2Z_1Z_2} = \sqrt{\frac{1}{4}(R + j\omega L)^2 + \frac{R + j\omega L}{G + j\omega C}} \qquad (9)$$

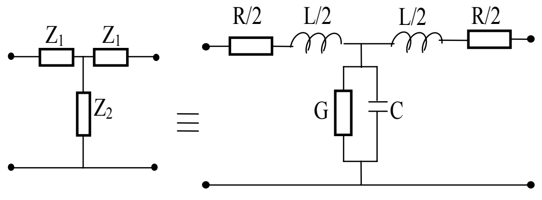

Fig. 3. The elementary network

The parameters R, L, G and C of the transmission line are uniformly distributed, resulting in that:

$$R=\Delta l R_L;\ L=\Delta l L_L;\ G=\Delta l G_L;\ C=\Delta l C_L \qquad (10)$$

where Δl is the length of the line segment and R_L, L_L, G_L and C_L are called line parameters and they have punctual value in line.

Relation (8) becomes:

$$l = \sqrt{\frac{\Delta l^2}{4}(R_L + j\omega L_L)^2 + \frac{R_L + j\omega L_L}{G_L + j\omega C_L}} \qquad (11)$$

$$l = Z_e \qquad (12)$$

When Δl → 0, it follows that:

$$Z_e = \sqrt{\frac{R_L + j\omega L_L}{G_L + j\omega C_L}} \qquad (13)$$

In microwave domain when ω is very large compared to R_L and G_L, it results in:

$$\lim_{\substack{\Delta l \to 0 \\ \omega \to \omega_{sup}}} Z_e = \lim_{\substack{\Delta l \to 0 \\ \omega \to \omega_{sup}}} \sqrt{\frac{\Delta l^2}{4}(R_L + j\omega L_L)^2 + \frac{\frac{R_L}{j\omega} + L_L}{\frac{G_L}{j\omega} + C_L}} = \sqrt{\frac{L_L}{C_L}} \qquad (14)$$

So,

$$Z_0 = \sqrt{\frac{L_L}{C_L}} \qquad (15)$$

where Z_0 is the characteristic impedance of the line.

3. THR WAVE PROPAGATION PATTERN

To highlight the phenomenon of the propagation of the voltage or current wave in a line segment, we consider a succession of two elementary networks (Morariu *et al.*, 2009:25) of filters in a portion of the line segment, as shown in fig. 4.

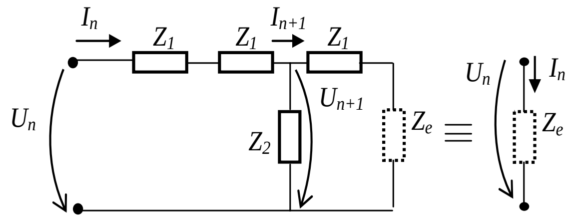

Fig. 4. Sequence of elementary networks

With representation in fig. 4, we establish the following relations:

$$U_n - U_{n+1} = I_n \cdot 2Z_1 \qquad (16)$$

$$I_n = \frac{U_n}{Z_e} \qquad (17)$$

$$U_n - U_{n+1} = U_n \frac{2Z_1}{Z_e} \qquad (18)$$

Respectively

$$\frac{U_n - U_{n+1}}{U_n} = \frac{2Z_1}{Z_e} \qquad (19)$$

Relation (19) defines the phenomenon of the voltage wave propagation in a transmission line.

Substituting Z_1, respectively Z_e with electrical parameters R, L, G and C we obtain:

$$\frac{U_n - U_{n+1}}{U_n} = \frac{R + j\omega L}{\sqrt{\frac{1}{4}(R + j\omega L)^2 + \frac{R + j\omega L}{G + j\omega C}}} \qquad (20)$$

Replacing the line parameters in equation (20) it results:

$$\frac{U_n - U_{n+1}}{U_n} = \\ = \frac{\Delta l(R_L + j\omega L_L)}{\sqrt{\frac{\Delta l^2}{4}(R_L + j\omega L_L)^2 + \frac{R_L + j\omega L_L}{G_L + j\omega C_L}}} \quad (21)$$

Given that $R_L + j\omega L_L$ has a finite value and $\Delta l^2/4$ tends to zero faster than Δl, we can neglect the influence of the expression $\frac{\Delta l^2}{4}(R_L + j\omega L_L)^2$ without changing the physical effect of the expression (21). Thus, we obtain:

$$\frac{U_n - U_{n+1}}{U_n} = \\ = \sqrt{\Delta l(R_L + j\omega L_L)(G_L + j\omega C_L)} \quad (22)$$

If variable l is associated with coordinate z from space (x, y, z) and $U_n - U_{n+1} = \Delta U_n$ the resulting formula is:

$$\frac{\Delta U_n}{U_n} = \Delta z\sqrt{(R_L + j\omega L_L)(G_L + j\omega C_L)} \quad (23)$$

As the index n is taken arbitrarily may be neglected, resulting in the expression:

$$\frac{\Delta U}{U} = \Delta z\sqrt{(R_L + j\omega L_L)(G_L + j\omega C_L)} \quad (24)$$

But ΔU and Δz is simultaneously at value of 0, and we can write:

$$\frac{dU}{U} = \Delta z\sqrt{(R_L + j\omega L_L)(G_L + j\omega C_L)} \quad (25)$$

Noting with

$$\gamma = \sqrt{(R_L + j\omega L_L)(G_L + j\omega C_L)} \quad (26)$$

we get the relation:

$$\frac{dU}{dz} = U\gamma \quad (27)$$

This relationship describes the overall distribution of the propagation phenomenon along a line and has the solution $U(z) = U_s e^{\gamma z}$, where U_s is the voltage at the load.

Derived, the result is

$$\frac{d^2U}{dz^2} = \frac{dU}{dz}\gamma = U\gamma^2 \quad (28)$$

This relation is a differential equation that describes the dynamics of the propagation phenomenon along transmission lines. Solving equation (28) shall be obtained voltage wave solutions that emphasize the simultaneity of the direct and inverse wave, known relationship of the propagation phenomenon.

$$U = Ae^{\gamma z} + Be^{-\gamma z} \quad (29)$$

Using equation (16) and the relation

$$Z_1 = \frac{1}{2}(R + j\omega L) \quad (30)$$

is determined following expression:

$$U_n - U_{n+1} = \Delta U_n = I_n(R + j\omega L) \quad (31)$$

But

$$(R_L + j\omega L_L)\Delta l = R + j\omega L \quad (32)$$

and $$\frac{\Delta U_n}{\Delta l} = I_n(R_L + j\omega L_L) \quad (33)$$

If l=z, using a Cartesian coordinates system (x, y, z), and any n, then:

$$\frac{\Delta U}{\Delta z} = I(R_L + j\omega L_L) \quad (34)$$

When $n \to \infty$ and $\Delta z \to 0$, the result is:

$$\frac{dU}{dz} = I(R_L + j\omega L_L) \quad (35)$$

Using relations number (26), (27) and (35) we obtain:

$$U\gamma = I(R_L + j\omega L_L) \quad (36)$$

and

$$U\sqrt{\frac{(G_L + j\omega C_L)}{(R_L + j\omega L_L)}} = I \quad (37)$$

For current waves following relations are established in accordance with the drawing of fig. 4:

$$I_n - I_{n+1} = \frac{U_{n+1}}{Z_2} = U_{n+1} Y_2 \quad (38)$$

$$I_n - I_{n+1} = U_{n+1}(G + j\omega C) \quad (39)$$

$$\frac{I_n - I_{n+1}}{\Delta l} = U_{n+1} \frac{(G + j\omega C)}{\Delta l} \quad (40)$$

$$\leftarrow (10) \rightarrow$$

$$\frac{I_n - I_{n+1}}{\Delta l} = \frac{\Delta I_n}{\Delta l} = U_{n+1}(G_L + j\omega C_L) \quad (41)$$

For l=z, using a Cartesian coordinates system (x, y, z), n→∞ and Δz→0, we get the relation:

$$\frac{dI}{dz} = U(G_L + j\omega C_L) \quad (42)$$

Taking into consideration the relation (27), it follows

$$\frac{d^2 I}{dz^2} = \frac{dU}{dz}(G_L + j\omega C_L) \quad (43)$$

and

$$\frac{d^2 I}{dz^2} = I\gamma^2 \quad (44)$$

The solution of the equation (44) expresses a second current wave propagation mod along over the line and is:

$$I = De^{\gamma z} + Ee^{-\gamma z} \quad (45)$$

It highlights current incident wave $De^{\gamma z}$ and reflected current wave $Ee^{-\gamma z}$.

3. CONCLUSIONS

The determination model presented uses recursive string theory applied to a transmission line structure. T-symmetric filters are assimilated to the punctual elements off the line. The propagation pattern obtained is very close to physical phenomena, allowing a complete understanding of the propagation in transmission lines.

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